Similarity Learning Over Large Collaborative Networks

Thèse n. XXXX 2013
présentée le 28 février 2013
à la Faculté Sciences et Techniques pour l'Ingénieur
Institut de Recherche Idiap
programme doctoral en Informatique et Communication (EDIC)
École Polytechnique Fédérale de Lausanne

ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

pour l'obtention du grade de Docteur ès Sciences
par

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soumise le 24 janvier 2013 au jury:

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Lausanne, EPFL, 2013

Acknowledgements

I would like to thank my advisers Andrei Popescu-belis and Herve Bourlard, for giving me the opportunity to work on this interesting topic. I would especially thank Andrei for his support, help and availability during the past four years.
I also would like to thank Ronan Collobert. I learned many machine learning concepts from him, and also he has made himself available for research discussions consistently. Many ideas in this thesis have their roots in these discussions.
I am thankful to my thesis committee - Karl Aberer, Samy Bengio, Jean-Cedric Chappelier, and Andreas Krause - for being part of my thesis and providing constructive feedbacks to improve it.
Interaction and discussion with Kate, Afsaneh, Gelareh, Marc, Gokul, Nikos, Darshan, Dinesh, Tatiana, Marco, Francesco, Nikolae, Leo, Mathew and many other Idiapers, was a great source of learning. I would like to thank Remi and Kenneth for making ICC a great experience. I thank Alex, Thomas and CC for making work more enjoyable.
Thanks to Maryam, Hassan and Mehdi who shared generously many good times with me. I would like to thank Samira for her consistent support and understanding that helped me to keep high standards alive inside me.
I would like to thank Marjan specially. Her constant encouragement, positive thinking and kindness over years brought life to my life and shaped my personality, and I am thankful for that. I also thank my parents who always supported my education, mentally and financially, and respected my choices.
I would like to thank my lifelong friend Mohsen Hosseini. Interaction with him over the years played a great role in shaping my critical thinking and research spirit.
I also would like to acknowledge my funding source, the Swiss National Center of Competence in Research (NCCR) on Interactive Multimodal Information Management (IM2).

Abstract

In this thesis, we propose novel solutions to similarity learning problems on collaborative networks. Similarity learning is essential for modeling and predicting the evolution of collaborative networks. In addition, similarity learning is used to perform ranking, which is the main component of recommender systems. Due to the the low cost of developing such collaborative networks, they grow very quickly, and therefore, our objective is to develop models that scale well to large networks.

The similarity measures proposed in this thesis make use of the global link structure of the network and of the attributes of the nodes in a complementary way. We first define a random walk model, named Visiting Probability (*VP*), to measure proximity between two nodes in a graph. *VP* considers all the paths between two nodes collectively and thus reduces the effect of potentially unreliable individual links. Moreover, using *VP* and the structural characteristics of small-world networks (a frequent type of networks), we design scalable algorithms based on *VP* similarity. We then model the link structure of a graph within a similarity learning framework, in which the transformation of nodes to a latent space is trained using a discriminative model. When trained over *VP* scores, the model is able to better predict the relations in a graph in comparison to models learned directly from the network's links.

Using the *VP* approach, we explain how to transfer knowledge from a hypertext encyclopedia to text analysis tasks. We consider the graph of Wikipedia articles with two types of links between them: hyperlinks and content similarity ones. To transfer the knowledge learned from the Wikipedia network to text analysis tasks, we propose and test two shared representation methods. In the first one, a given text is mapped to the corresponding concepts in the network. Then, to compute similarity between two texts, *VP* similarity is applied to compute the distance between the two sets of nodes. The second method uses the latent space model for representation, by training a transformation from words to the latent space over *VP* scores. We test our proposals on several benchmark tasks: word similarity, document similarity / clustering / classification, information retrieval, and learning to rank. The results are most often competitive compared to state-of-the-art task-specific methods, thus demonstrating the generality of our proposal. These results also support the hypothesis that both types of links over Wikipedia are useful, as the improvement is higher when both are used.

In many collaborative networks, different link types can be used in a complementary way.

Therefore, we propose two joint similarity learning models over the nodes' attributes, to be used for link prediction in networks with multiple link types. The first model learns a similarity metric that consists of two parts: the general part, which is shared between all link types, and the specific part, which is trained specifically for each type of link. The second model consists of two layers: the first layer, which is shared between all link types, embeds the objects of the network into a new space, and then a similarity is learned specifically for each link type in this new space. Our experiments show that the proposed joint modeling and training frameworks improve link prediction performance significantly for each link type in comparison to multiple baselines. The two-layer similarity model outperforms the first one, as expected, due to its capability of modeling negative correlations among different link types.

Finally, we propose a learning to rank algorithm on network data, which uses both the attributes of the nodes and the structure of the links for learning and inference. Link structure is used in training through a neighbor-aware ranker which considers both node attributes and scores of neighbor nodes. The global link structure of the network is used in inference through an original propagation method called the Iterative Ranking Algorithm. This propagates the predicted scores in the graph on condition that they are above a given threshold. Thresholding improves performance, and makes a time-efficient implementation possible, for application to large scale graphs. The observed improvements are explained considering the structural properties of small-world networks.

Keywords: Distance Metric Learning, Similarity Learning, Social Network Analysis, Collaborative Network Analysis, Learning to Rank, Link Prediction, Text Semantic Similarity, Random Walk, Transfer Learning, Classification, Clustering.

Résumé

Apprentissage de mesures similarité sur des graphes collaboratifs de grande taille

Dans cette thèse, nous proposons de nouvelles solutions au problème de l'apprentissage automatique de mesures de similarité appliquées aux réseaux ou graphes collaboratifs. L'apprentissage des mesures de similarité est essentiel pour modéliser et prédire l'évolution des réseaux collaboratifs. De plus, ces mesures sont utilisées pour des tâches de classement qui sont la composante principale des systèmes de recommandation. Le faible coût de création de ces réseaux collaboratifs fait qu'ils croissent très vite. C'est pourquoi un de nos objectifs est de proposer des modèles applicables à des réseaux de grande taille.

Les mesures de similarité proposées dans cette thèse utilisent la structure globale des liens du graphe et les attributs des nœuds d'une façon complémentaire. Nous définissons d'abord un modèle de marche aléatoire appelé Probabilité de Visite (*VP* en anglais) pour mesurer la proximité de deux nœuds dans un graphe. Le modèle considère l'ensemble des chemins entre deux nœuds, ce qui réduit l'effet des liens individuels, potentiellement peu fiables. De plus, partant de la *VP* et des caractéristiques des réseaux de type "petit monde" (un type relativement fréquent), nous proposons des algorithmes adaptés aux graphes de grande taille. Nous modélisons ensuite la structure de liens dans un cadre permettant l'apprentissage discriminatif de la similarité, qui projette les nœuds dans un espace latent. Lorsqu'il est entraîné sur les valeurs de *VP*, ce modèle fait de meilleurs prédictions sur les liens qu'un modèle entraîné directement sur les liens du graphe.

Utilisant toujours l'approche *VP*, nous expliquons comment transférer les informations contenues dans une encyclopédie hypertexte pour les appliquer à des tâches d'analyse de textes. Nous utilisons le graphe de Wikipedia avec deux types de liens entre articles : les hyperliens d'origine et des liens construits à partir de la similarité lexicale. Afin de transférer l'information acquise à partir de Wikipedia vers les tâches d'analyse de texte, nous proposons deux modèles de représentation. Dans le premier, un texte est mis en correspondance avec les concepts les plus pertinents du graphe, puis, pour calculer la similarité entre deux textes, nous utilisons la *VP* entre les ensembles de concepts. Dans le second modèle, un espace latent sert de représentation commune, et une fonction de transformation entre les mots et l'espace latent est entraînée avec la *VP* comme critère. Ces propositions sont testées sur plusieurs tâches : similarité de mots, similarité de documents, partitionnement de documents, classification de documents, recherche d'information, et apprendre à classer. Les résultats sont le plus souvent comparables aux meilleurs résultats des méthodes conçues spécifiquement pour chaque tâche, ce qui démontre la généralité de notre modèle. De plus, les résultats montrent que les deux types de liens sont utiles.

Les différents types de liens existant dans les graphes collaboratifs peuvent souvent être utilisés de manière complémentaire. Nous proposons deux modèles pour l'apprentissage de la similarité sur les attributs des nœuds, à utiliser pour la prédiction de liens dans les graphes avec plusieurs types de liens. Le premier modèle apprend une mesure de similarité avec deux composants : la partie générale, partagée par tous les types de liens, et la partie spécifique, entraînée de manière séparée sur chaque type. Le second modèle comprend deux couches : la première (partagée par tous les types de liens) projette les nœuds dans un nouvel espace, puis une fonction de similarité est entraînée pour chaque type de liens dans cet espace. Nos expériences montrent que ces modèles améliorent la prédiction de liens pour chaque type. Le modèle à deux couches est, comme prévu, meilleur que le premier, grâce à sa capacité à utiliser aussi des corrélations négatives entre types de liens.

Nous proposons, en final, un algorithme pour apprendre à classer les nœuds, qui utilise à la fois les attributs des nœuds et la structure des liens. Cette dernière est utilisée grâce à une méthode de propagation originale appelée Algorithme Itératif de Classement. Cette méthode propage les scores prédits à travers le graphe, à condition qu'ils dépassent un certain seuil. L'utilisation d'un seuil améliore les scores et aboutit à un algorithme rapide pouvant être appliqué aux graphes de grande taille. Les améliorations observées sont analysées dans la perspective des propriétés structurelles des graphes du type "petit monde".

Mots-clés : Apprentissage de distances sur les graphes, Apprentissage de mesures de similarité, Analyse des réseaux sociaux, Analyse de réseaux collaboratifs, Apprendre à classer, Prédiction de liens, Similarité sémantique de textes, Marche aléatoire, Transfert de l'apprentissage, Classification, Partitionnement.

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1 Introduction

Collaborative online systems make it easy to generate large public networks of data objects. The cost of creating a new object or a new link between two objects is very small, and is usually contributed by users. This low cost explains that such collaborative networks grow very quickly, easily reaching networks of millions of objects based on the collaboration of as many users. The Web, Wikipedia, social bookmarks, product preference networks, research articles with citations, and online social networks are examples of such collaborative networks which shape our daily life.

Moreover, new networks can be inferred from original collaborative ones. For example, consider the network of product preferences in an online store, in which the preference of a customer for a product is represented by a link, with its strength being coded by the links' weight. We can infer from this network how much two products are co-preferred and therefore, the network showing the co-preferred relations between the products in the store can be built from the original network. Co-authorship is another network of this type. We call this type of networks collaborative networks as well, because they are inferred directly from collaborative networks and evolve similarly.

In such collaborative systems, there is usually no central editing authority. Every user acts autonomously when creating new content. Therefore, not all content in these networks is equally reliable. The content related to less popular objects is likely to be less reliable. Conversely, content becomes more and more reliable as it is shaped by the collaboration of more and more users.

Hence, designing robust methods to perform prediction and analysis of such networked systems is a valuable focus of interest. Prediction of a system's evolution can be used to make recommendations, which can *facilitate* and also *improve* the formation of the network.

For example, consider a network with scientific papers and citations. A recommender system can facilitate the identification of related articles by authors, which is known to be a difficult and time-consuming task when writing research articles. Moreover, such a system can help authors to find articles that would have been missed otherwise. This can happen for various reasons such as the low popularity of an article, recency, or lack of knowledge of the authors. In the latter case, a recommender system would not only facilitate the formation of the network, but also improve it.

The main component of a recommender system is its ranking functionality. Recommender systems, given either an implicit or an explicit query, rank all objects, and almost always return only the top ranked ones. For example, in an online store, a recommender system would recommend top co-preferred items with respect to a specific item (an implicit query) and thus help customers to find the most suitable item for their needs and preferences. Besides, a different recommender system could return top ranked items for a keyword entered by a customer (explicit query). In both cases, for explicit or implicit queries, the main task is to rank objects from a network with respect to the query object.

Ranking objects based on a query can be performed by defining a distance measure between objects and sorting them based on their distance to the query object. Similarly, link formation in social networks is more likely if two nodes have similar social characteristics – a phenomenon known as homophily in social networks [McPherson et al., 2001]. Therefore, the problem of ranking, or even more generally the modeling of collaborative networks, can be transformed into the problem of finding a distance or similarity measure over the networks.

Several sources of information should be considered when building a distance measure over a collaborative network. The first important source of information is the global link structure of the network. Collaborative networks typically contain many spurious links, or are incomplete, making a proximity measure based on local link structure unreliable. Therefore, a robust proximity measure based on global link structure needs to be designed, in particular so that it can be applied to large networks. The link structure might consist of multiple link types. Also, the links can be weighted: for example, in a network of product co-preference, weighted links indicate how much two products are co-preferred. Therefore, similarity learning over collaborative networks should be able to consider in a robust way these additional sources of information.

Another source of information are the attributes of the objects. These can be used to model the networks' evolution and consequently the distance between objects. For example, in a paper/citations network, the similarity of the articles can be modeled by their content words. Content words can improve the accuracy of the model, especially when the link structure is not very rich, for example in case of missing citations, low-popularity papers, or either very new or very old papers.

In addition to the prediction task in collaborative networks, the analysis of the evolution of a network with respect to node attributes, provides valuable insights that can be used in designing methods which aim to maximize another utility function. For example, consider the hypothetical situation in which an online store manager wants to choose a title for her new product among some different candidate titles. A model that relates the keywords in the title to the likelihood of a sale can guide the manager to optimize the choice of keywords.

In general, the attributes of the nodes are not sufficient to model the network. For instance, let us consider a product co-preference network in which the products' attributes are derived from the title of each product along with category information. The title is a short description of the product that only makes sense when interpreted by humans and would likely not be sufficient to model the network. Moreover, feature selection methods are not loss free. For example in the case of a papers/citations network, if the content of a paper is represented by a bag of keywords, these cannot grasp the entire content of the paper. Besides, usually some attributes are missing. For example in the case of online social networks, people tend not to fill in all the information fields in their profile. Hence, nodes' attributes and global network structure (relational attributes) should be used in a complementary way.

The overall contribution of this thesis is two-fold. First, we propose distance learning methods over **large** and **collaborative** networks with **high-dimensional** features, to be used for prediction tasks which make use of the global structure of a network and the attributes of its nodes. Second, we show how to leverage reliable content in these networks to perform text analysis tasks, improving the state-of-the-art performance in situations when only a small quantity of labeled data is available. The following sections of this introduction summarize the achievements corresponding to the diffeent chapters of the thesis.

Symmetric Random Walk on Large Graphs: Approximation Algorithms and Similarity Learning

This chapter consists of two main parts, in the first part we define a random walk model, Visiting Probability (*VP*), to measure proximity between two nodes in a graph. *VP* considers all the paths between two nodes collectively and thus reduces the effect of unreliable links. We define symmetric *VP* similarity measure and show that using the symmetric *VP* improves the prediction performance of the distance.

Moreover, we show how to make use of *VP* definition and design approximation algorithm to perform ranking based on *VP* on large graphs. Besides, we define community of a node according to *VP* and bring experimental evidences that the community definition is effective. Fast algorithms are designed to solve K-nearest neighbors and community identification over large graphs based on symmetric *VP* proximity.

A small-world network is a type of graph in which most nodes are not neighbors of one another, but most nodes can be reached from every other by a small number of hops or steps. It has been shown that many real-world complex systems can be modeled by small-world networks including but not limited to neuronal networks, food webs, social networks, scientific-collaboration networks and World Wide Web. We make use of this structural property to design fast algorithms on collaborative networks, we use this structural property again in Chapter 6.

In the second part of this chapter, we show that any relation between two nodes in a graph can be interpreted as the proximity between those two nodes in a latent space. Therefore, the link structure of a graph can be modeled by a similarity learning framework in which the transformation of nodes to the latent space is trained using a discriminative model. We show how to apply this framework to learn the similarity between two nodes based on the node's attributes over large graphs. Moreover, we show that similarity learning on attributes from random walk scores, specifically *VP* scores here, can model and predict better the relations in the graph in comparison to learning on the network's links directly. Therefore, we use both global network structure (through using *VP* scores) and node attributes to learn a reliable similarity measure. At the end we evaluate the effectiveness of the proposed models on link prediction task on various networks. We show experimentally that if the node attributes are not predictive enough, ignoring the global link structure for inference can reduce the prediction performance, we address this issue in Chapter 6.

Transfer Learning from Hypertext Encyclopedia to Text Analysis Tasks

In this chapter, we explain how to transfer knowledge from a hypertext encyclopedia to text analysis tasks. The *VP* proximity is used in this chapter to compute semantic relatedness between words or texts (sets of words), by taking advantage of content-based and link-based knowledge from hypertext encyclopedias such as Wikipedia.

A network of concepts is first constructed by filtering the encyclopedia's articles, each concept corresponding to an article. Two types of weighted links between concepts are considered: one based on hyperlinks between articles, and another one based on the lexical similarity between them. A given text is mapped to the corresponding concepts in this network and then to compute similarity between two texts *VP* similarity is applied to compute the distance between sets of nodes.

Moreover, we analyze the convergence of the approximation methods proposed in chapter 3, over the English Wikipedia data set, and measure various characteristics of the network to help understanding its usefulness for text analysis tasks.

Finally, to make the algorithm tractable and also make transfer learning possible to other machine learning tasks and algorithms, we use the latent space model which we explained in the chapter 3 to train a transformation from words to a latent space over *VP* scores. Therefore, we can add one layer to other text analysis tasks which applies this transformation to the words and insert Wikipedia knowledge to the task.

To evaluate the proposed distance, we apply our method to four important tasks in natural language processing: word similarity, document similarity, document clustering, and document classification, along with a ranking task for information retrieval. The performance of our method is state-of-the-art or close to it for all the tasks, thus demonstrating the generality of the method and the accompanying knowledge resource. Moreover, we show that using both hyperlinks and lexical similarity links improves the scores with respect to a method using only one of them, because hyperlinks bring additional real-world knowledge not captured by lexical similarity. In Chapter 5 we design algorithms to make use of different link types more formally.

Additionally, the proposed method is applied to Idiap's just-in-time information retrieval system (ACLD), and brings improvement in terms of the relevance of results.

Joint Similarity Learning for Predicting Links in Multi Links Networks

This chapter addresses the problem of link prediction on large multi-link networks (i.e. with links of multiple types) by proposing two joint similarity learning architectures over the attributes of the nodes. The first model is a similarity metric that consists of two parts: a general part, which is shared between all link types, and a specific part, which learns the similarity for each link type specifically.

The second model consists of two layers: the first one, which is shared between all link types, embeds the objects of the network into a new space, while the second one learns the similarity between objects for each link type in this new space. The similarity metrics are optimized using a large-margin optimization criterion in which connected objects should be closer than non-connected ones by a certain margin. A stochastic training algorithm is proposed, which makes the training applicable to large networks with high-dimensional feature spaces.

The models are tested on link prediction for two data sets with two types of links each: TED talks and Amazon products. The experiments show that jointly modeling of the links given our frameworks improve link prediction performance significantly for each link type. The improvement is particularly higher when there are fewer links available from one link type in the network. Moreover, we show that transfer learning from one link type to another one is possible using the above frameworks.

At the end, we show how this model can be generalized to similar but different tasks, including joint classification and link prediction, and transfer learning between networks with (approximately) the same set of attributes.

Similarity Learning for Collective Ranking on Networks: Application to Link Prediction

Traditionally in learning to rank approaches data points are assumed to be independent. Although this assumption leads to acceptable results in many applications, it is quite questionable when dealing with network data. Moreover, methods based on the link structure including methods based on common neighbors and random walk models are ignorant about the nodes' attributes in the networks.

In this chapter, a method is proposed for learning to rank on network (relational) data, which makes use of the features of the nodes as well as the existing links between them. First, a neighbors-aware ranker is trained using a pairwise loss function. Then, collective inference is performed using a sparse iterative ranking algorithm, which propagates the results of rankers over the network.

The method is applied to three data sets with papers/citations and webpages/hyperlinks. The results show that the proposed algorithm, using both link structure and node attributes, outperforms several other methods: a content-only ranker, a link-only one, an unsupervised random walk method, a relational topic model, and a method based on the weighted numbers of common neighbors. In addition, the propagation algorithm improves results even when no prior link structure is known, and scales efficiently to large networks.

Conclusion and Future Work

In the final chapter, we first summarize the achievements of the previous chapters, mainly in designing distance learning methods over large collaborative networks with high-dimensional features, and demonstrating their relevance on a significant number of prediction tasks. Moreover, we summarize the achievement of leveraging reliable content from these networks to text analysis tasks, which has improved the state-of-the-art performance in situations when only a small quantity of labeled data is available.

The focus of this work was on learning a similarity function between two objects. But almost always in recommender systems a set of top items are shown as a result and therefore, one should rather aim at an objective function that assigns a score to sets of retrieved objects as a whole. In preparation for future work, we define formally the concepts of *consistency* and *diversity*, and relate them to the models we presented in this thesis. The main desirable properties of such a global objective function are: first, it should give a higher score to a set of items that are *related* to the query object, and second, it should emphasize either the *diversity* or the *consistency* of the items. For large data sets, finding a set with a maximal score is not tractable, but we have found that greedy algorithms may help to overcome this problem, and we foresee further investigations on this topic.

2 Related work

The overall focus of this thesis, as presented earlier in the introduction, is two-fold. The first contribution consists of new distance learning methods over **large** and **collaborative** networks with **high-dimensional** features, to be used for prediction tasks that use the global network structure and the node attributes. As mentioned, distance metric learning usually happens in a ranking framework. Traditionally, distance metric learning and learning to rank methods were designed for non-relational data sets. In this thesis, we generalize these methods to be applicable to large relational data sets. We also discuss in this chapter and the following ones, previous methods which are using only the link structure of a network to infer the distance, and thus ranking. We show how the methods in this manuscript evolved in comparison to those methods.

Therefore, the related work corresponding to the first contribution of the thesis falls into several categories: (1) learning to rank and distance metric learning; (2) link prediction; (3) distance and ranking based on link structure in networks. In this chapter, we discuss each category.

Moreover, we show how to leverage reliable content in hypertext encyclopedias (mainly Wikipedia in this thesis) to achieve better text semantic similarity measures, which are an essential part of many text analysis tasks. Therefore, related work for this part spans a large number of domains and approaches, and can be divided mainly into: (1) previous methods for computing semantic relatedness, including uses of Wikipedia or other networked resources, for one or more tasks in common with the tasks discussed here; (2) state-of-the-art methods and scores for each of the targeted tasks. In fact, many combinations of tasks, methods and resources may share one or more elements with our work. In this chapter, we will focus on the first type of previous work, while for the other one, performance comparisons with state-of-the-art methods will be made in each of the application sections of the subsequent chapters.

2.1 Distance Metric Learning and Learning to Rank

Many methods for learning to rank have been proposed, with different leaning abilities. We discuss here some of the main works in this field and show how the proposals in this thesis relate to them.

Perception Ranking [Crammer and Singer, 2001] is an online algorithm for ordinal classification. The algorithm can be employed for ranking as a pointwise method. The main idea is to learn several parallel perceptron models which make classification between the neighboring grades. Following a pointwise approach, the ranker is trained based on the grade of each object, whereas in a pairwise method the ranker is trained on the order (rank) of the pair of objects. In our work, we always use a pairwise training method, which was shown to be more effective than pointwise methods [Li, 2011].

IR SVM [Cao et al., 2006] is a pairwise method which formulates the ranking problem as an SVM classification, and adapts this to the document retrieval problem. The feature selection that transforms the ranking problem into a classification one – building features for classification from the query and each target document – is not effective on all data sets. This is especially problematic for a task such as link prediction, where we have many non-linked examples. Similarly, SVMRank [Joachims, 2002] is a pairwise ranking algorithm which transforms the pairwise ranking to SVM binary classification. For SVMRank, batch optimization on large graphs is not possible considering the number of non-connected pairs. In comparison to these approaches, we adapt the training algorithm to be applicable to large graphs with high-dimensional features. Moreover, we generalize the methods in a way that can use the network's structural features, and can efficiently model multi edge networks.

Grangier and Bengio [2008] introduce a discriminative model for the retrieval of images from text queries using a learning procedure optimizing a ranking criterion. The proposed model addresses the retrieval problem directly and does not rely on an intermediate image annotation, and the training procedure is based on the online learning of kernel-based classifiers, and therefore is scalable to large data sets.

RankNet [Burges et al., 2005] is a feed forward neural network which is trained by back propagation to learn the scores of each training example. In a similar vein, Bai et al. [2010] perform supervised training of a nonlinear model over vectors of words, to preserve a pairwise ranking between documents. Their approach scales well to large data sets with a large number of words. Similarly, Shaw et al. [2011] train a distance metric by stochastic gradient descent over a hinge loss function that preserves the network structure. Weston et al. [2011] propose a faster training algorithm to learn a low-dimensional joint embedding space for both images and annotations which optimizes precision at the top of the ranked list of annotations for a given image. We will follow the same general line to build our ranking methods based on similarity learning, by keeping in mind that the framework should be applicable to large graphs.

A large margin nearest neighbor classifier is built by training a Mahalanobis distance metric by

Weinberger et al. [2006]. Similarly, in Section 4.9, we build a distance learning classifier which learns, given a training set, a similarity measure in a latent space so that for each data point in the training set, its similarity to data points with the same label (or class) is higher than the similarity to data points with different labels. We show that prior knowledge leaned from the Wikipedia network can be successfully transferred to this classifier.

In summary, in comparison to the conventional learning to rank approaches which assume that data points are independent, we consider dependency between network's objects by using neighborhood information as well as object features in the ranker (Chapter 6 and Section 3.4). In addition, we allow different link-type rankers sharing information among them and model more accurately the networks' structure (Chapter 5).

2.2 Supervised Link Prediction

Link prediction can be formulated as a supervised learning task in which the goal is to predict new link formation. Backstrom and Leskovec [2011] describe the problem of link prediction as a supervised learning task and illustrate how a method known as supervised random walks can address the link prediction task. This overcomes one of the main shortcomings of the link-based previous works by using attribute information to make predictions. However, this method requires to compute the gradient iteratively in each iteration, which makes the training inefficient. Supervised random walk (similarly to link-based methods) is not able to perform ranking when the query object is not part of the network, whereas our pairwise approaches, trained on the query and target nodes' attributes, can perform ranking in this situation.

Similarly, Agarwal et al. [2006] propose a supervised learning method for ranking the objects in a graph. Their method uses a random walk model and learns the transition probabilities from the ordered pairs of objects in the training data. A transition probability is learned for each link, which makes overfitting likely and is not effective for large-scale graphs with many edges, given that there are many parameters to learn. Similarly to supervised random walk, this method is not applicable when the query object is not part of the network.

Miller et al. [2009] adopt a generative Bayesian nonparametric approach to simultaneously infer the number of latent features and to learn which entities have each feature. The method is difficult to train, and inference for large scale graphs is time-consuming, whereas we consider designing methods applicable to large networks.

Relational Topic Models (RTM) [Chang, 2009] consider both the documents and the links between them. For each pair of documents, an RTM models their link as a binary random variable that is conditioned on their contents. The model can be used to predict links between documents and is used as one of our baselines (see Section 6.4). The inference and learning algorithms are based on variational methods. Our proposed method in Chapter 6 outperforms RTM on the studied networks by a large margin. The neighborhood information is not modeled explicitly in RTM in comparison to our method.

Menon and Elkan [2011] propose a link prediction algorithm based on the matrix factorization. The algorithm essentially is similar to the latent space model which we explain in Section 3.4. We show in Section 3.4 that learning over random walk scores, instead of networks' edges, improve the predictability of the model.

In the case of multi-preference data, also called multiple link type or multi-link networks, Paccanaro and Hinton [2000] proposed Linear Relational Embeddings in which entities are embedded in a latent space and relations in this latent space are modeled by linear operations. This idea has been further improved in the Structured Embeddings (SE) framework proposed by Bordes et al. [2011]. We follow the same path and investigate further the modeling of multi preference data in Chapter 5 of this thesis.

In summary, we transform link prediction problem to a ranking problem in which given the query node, other nodes are ranked based on the likelihood of forming a link with the query. We will propose methods using the objects' attributes and the relations between objects in a network in an effective way to perform ranking, applicable to large networks. In comparison to supervised generalizations of random walk models [Backstrom and Leskovec, 2011, Agarwal et al., 2006], our model can be applied even when the query object does not have any known links in the network. In this case, the performance can be improved by modeling the links between the rest of the objects in the network, as we propose with the model described in Chapter 6.

2.3 Ranking and Distance Based on Link Structure

The link prediction task can be formulated as a ranking task of pairs of nodes, using link structure similarity metrics, for example random walk metrics or metrics using the common neighborhood. The Adamic and Adar [2001] similarity measure, which is based on common neighborhood, yields relatively high performance in link prediction [Liben-Nowell and Kleinberg, 2003].

It is well-known that estimating proximity in networks as the length of shortest path does not take into account the relative importance of these paths with respect to the overall properties of the network, such as the number and length of all possible paths between two nodes. The length of the shortest path is quite sensitive to spurious links. It has been shown that aggregated measures based on random walks are more effective for link prediction than individual links and paths [Brand, 2005, Sarkar and Moore, 2007, Liben-Nowell and Kleinberg, 2003].

Two popular random walk measures which are well studied in the previous works are hitting time, a standard notion in graph theory, and Personalized PageRank (PPR) [Haveliwala, 2003], surveyed by Berkhin [2005].

Hitting time has been used in several studies as a distance measure in graphs, e.g. for dimensionality reduction [Saerens et al., 2004] or for collaborative filtering in a recommender system [Brand, 2005]. Hitting time has also been used for link prediction in social networks along with other graph-based distances [Liben-Nowell and Kleinberg, 2003], or for semantic query suggestion using a query/URL bipartite graph [Mei et al., 2008]. A branch-and-bound approximation algorithm has been proposed to compute a node neighborhood for hitting time in large graphs [Sarkar and Moore, 2007, Sarkar et al., 2008].

We will develop a random walk model to measure the proximity between two nodes based on the networks' link structure in Chapter 3. We explain the relation between the popular random walk models above and the proposed random walk in Section 3.1.5. Moreover, we show how to use the definition of our measure to design fast algorithms applicable to large networks in Sections 3.2, 3.3.1 and 3.3.2.

A shortcoming of the state-of-the-art methods cited above is that they are not trained on node and edge attributes. Therefore, if the prior link structure around a query node is not known, then these methods can not perform well in finding similar nodes, e.g. candidates for linking. For example, in the case of paper citation, unless the paper contains already many citations, these methods can not perform well. To overcome this problem, we make a connection between random walk scores and a latent space model learned on the nodes' attributes in Chapter 3. Moreover, in Chapter 6, our proposed model overcomes this issue by using nodes' attributes in addition to the link structure to learn and infer a similarity function.

2.4 Word Semantic Relatedness using Graphs: WordNet and Wikipedia

Many graph-based methods have been applied to NLP problems and were recently surveyed by Navigli and Lapata [2010] with an application to word sense disambiguation. For instance, Navigli [2008] defined a method for truncating a graph of WordNet senses built from input text, while Navigli and Lapata [2010] focused on measures of connectivity and centrality of a graph built on purpose from the sentences to disambiguate, and are therefore close in spirit to the ones used to analyze our large Wikipedia-based network in Section 4.3.3.

PageRank has been used for word sense disambiguation over a graph derived from the candidate text by Navigli and Lapata [2010]. As for Personalized Page Rank (PPR), the measure has been used for word sense disambiguation by Agirre and Soroa [2009] over a graph derived from WordNet, with up to 120,000 nodes and 650,000 edges. PPR has also been used for measuring lexical relatedness of words in a graph built from WordNet by Hughes and Ramage [2007].

Many other attempts have been made in the past to define word and text similarity distances, for various applications to language technology. One approach is to construct – manually or semi-automatically – a taxonomy of word senses or of concepts, with various types of relations, and to map the text fragments to be compared onto the taxonomy. For instance, WordNet [Fellbaum, 1998] and Cyc [Lenat, 1995] are two well-known knowledge bases, respectively of word senses and concepts, which can be used for overcoming the strong limitations of pure lexical matching. A thesaurus such as Roget's can also be used for similar purposes [Jarmasz, 2003, Jarmasz and Szpakowicz, 2003]. This approach makes use of explicit senses or concepts that humans can understand and reason about, but the granularity of knowledge representation is limited by the taxonomy. Building and maintaining these knowledge bases requires a lot of time and effort from experts. Moreover, they may cover only a fraction of the vocabulary of a language, and usually include few proper names, conversational words, or technical terms.

Several methods for computing lexical semantic relatedness exploit the paths in semantic networks or in WordNet, as surveyed by Budanitsky and Hirst [2006, Section 2]. Distance in the network is one of the obvious criteria for similarity, which can be modulated by the type of links [Rada et al., 1989] or by local context, when applied to word sense identification [Leacock and Chodorow, 1998]. Resnik [1995, 1999] improved over distance-based similarity by defining the information content of a concept as a measure of its specificity, and applied the measure to word sense disambiguation in short phrases. An information-theoretic definition of similarity, applicable to any entities that can be framed into a probabilistic model, was proposed by Lin [1998] and was applied to word and concept similarity. This work and ours share a similar concern – the quest for a generic similarity or relatedness measure – albeit in different conceptual frameworks – probabilistic vs. hypertext encyclopedia.

Other approaches make use of unsupervised methods to construct a semantic representation of words or of documents by analyzing mainly co-occurrence relationships between words in a corpus (see e.g. Chappelier [2012] for a review). Latent Semantic Analysis [Deerwester et al., 1990] offers a vector-space representation of words, which is grounded statistically and is applied to document representation in terms of topics using Probabilistic LSA [Hofmann, 1999] or Latent Dirichlet Allocation [Blei et al., 2003]. These unsupervised methods construct a low-dimensional feature representation, or concept space, in which words are no longer supposed to be independent. The methods offer large vocabulary coverage, but the resulting "concepts" are difficult for humans to interpret [Chang et al., 2009].

Mihalcea et al. [2006] compared several knowledge-based and corpus-based methods (including for instance [Leacock and Chodorow, 1998]) and then used word similarity and word specificity to define one general measure of text semantic similarity. Results of several methods and combinations are reported in their paper. Because it computes word similarity values between all word pairs, the proposed measure appears to be suitable mainly for computing similarity between short fragments – otherwise, the computation becomes quickly intractable.

One of the first methods to use a graph-based approach to compute word relatedness was proposed by Hughes and Ramage [2007], using Personalized PageRank (PPR) [Haveliwala, 2003] over a graph built from WordNet, with about 400,000 nodes and 5 million links. Their goal (as ours) was to exploit all possible links between two words in the graph, and not only the shortest path. They illustrated the merits of this approach on three frequently-used data sets of word pairs – which will be also used in this thesis, see Section 4.6 – using several standard correlation metrics as well as an original one, and their scores were close to human inter-annotator agreement values.

In recent years, Wikipedia has appeared as a promising conceptual network, in which the relative noise and incompleteness due to its collaborative origin is compensated for by its large size and a certain redundancy, along with availability and alignment in several languages. Several large semantic resources were derived from it, such as a relational knowledge base (DBpedia [Bizer et al., 2009]), two concept networks (BabelNet [Navigli and Ponzetto, 2010] and WikiNet [Nastase et al., 2010]) and an ontology derived from both Wikipedia and WordNet (Yago [Suchanek et al., 2008]).

WikiRelate! [Strube and Ponzetto, 2006] is a method for computing semantic relatedness between two words by using Wikipedia. Each word is mapped to the corresponding Wikipedia article by using the titles. To compute relatedness, several methods are proposed, namely, using paths in the Wikipedia category structure, or using the contents of the articles. Our method, by comparison, also uses the knowledge embedded in the hyperlinks between articles, along with the entire contents of articles. Recently, the category structure exploited by WikiRelate! was also applied to computing semantic similarity between words [Ponzetto and Strube, 2011]. Overall, however, WikiRelate! measures relatedness between two words and is not applicable to similarity of longer fragments, unlike our method described in Chapter 4. Another method to compute word similarity was proposed by Milne and Witten [2008a] using similarity of hyperlinks between Wikipedia pages.

2.5 Text Semantic Relatedness

Several studies have measured relatedness of sentences or entire texts (a summary appears in Table 2.1 on page 15). In a study by Syed et al. [2008], Wikipedia was used as an ontology in three different ways to associate keywords or topic names to input documents: either (1) by cosine similarity retrieval of Wikipedia pages, or (2) by spreading activation through the Wikipedia categories of these pages, or (3) by spreading activation through the pages hyperlinked with them. The evaluation was first performed on three articles for which related Wikipedia pages could be validated by hand, and then on 100 Wikipedia pages, for which the task was to restore links and categories (similarly to [Milne and Witten, 2008b]). The use of a private test set makes comparisons with other work uneasy. In another text labeling task, Coursey et al. [2009] have used the entire English Wikipedia as a graph (5.8 million nodes, 65 million edges) with a version of Personalized PageRank [Haveliwala, 2003] that was initialized with the Wikipedia pages found to be related to the input text using Wikify! [Mihalcea and Csomai, 2007]. The method was tested on a random selection of 150 Wikipedia pages, with the goal of retrieving automatically their manually-assigned categories.

Ramage et al. [2009] have used Personalized PageRank over a WordNet-based graph to detect paraphrases and textual entailment. They formulated a theoretical assumption similar to ours: "the stationary distribution of the graph [random] walk forms a 'semantic signature' that can be compared to another such distribution to get a relatedness score for texts."

Explicit Semantic Analysis (ESA), proposed by Gabrilovich and Markovitch [2007, 2009], instead of mapping a text to a node or a small group of nodes in a taxonomy, maps the text to the entire collection of available concepts, by computing the degree of affinity of each concept to the input text. ESA uses Wikipedia articles as a collection of concepts, and maps texts to this collection of concepts using a term/document affinity matrix. Similarity is measured in the new concept space. Unlike our method, ESA does not use the link structure or other structured knowledge from Wikipedia. Our method, by walking over a content similarity graph, benefits in addition from a non-linear distance measure according to word co-occurrences.

ESA has been used as a semantic representation (sometimes with modifications) in other studies of word similarity, such as a cross-lingual experiment with several Wikipedias by Hassan and Mihalcea [2009], evaluated over translated versions of English data sets (see Section 4.6 below). In a study by Zesch et al. [2008], concept vectors akin to ESA and path length were evaluated for WordNet, Wikipedia and the Wiktionary, showing that the Wiktionary improved over previous methods. ESA also provided semantic representations for a higher-end application to cross-lingual question answering [Cimiano et al., 2009], and was used by Yeh et al. [2009], to which we now turn.

Probably the closest antecedent to our study is the WikiWalk approach [Yeh et al., 2009]. A graph of documents and hyperlinks was constructed from Wikipedia, then the Personalized PageRank (PPR) [Haveliwala, 2003] was computed for each text fragment, with the teleport vector being the one resulting from ESA. A dictionary-based initialization of the PPR algorithm was studied as well. To compute semantic similarity between two texts, Yeh et al. simply compared their PPR vectors. Their scores for word similarity were slightly higher than those obtained by ESA [Gabrilovich and Markovitch, 2009], while the scores on document similarity (Lee data set, see Section 4.7 below) were "well below state of the art, and show that initializing the random walk with all words in the document does not characterize the documents well." By comparison, in our method, we also consider in addition to hyperlinks the effect of word co-occurrence between article contents, and use a different random walk and initialization methods.

Mihalcea and Csomai [2007] and Milne and Witten [2008b] discussed enriching a document with Wikipedia articles. Their methods can be used to add explanatory links to news stories or educational documents, and more generally to enrich any unstructured text fragment (or bag-of-words) with structured knowledge from Wikipedia. Both perform disambiguation for all n-grams, which requires a time-consuming computation of relatedness of all senses to the context articles. The first method detects linkable phrases and then associates them to the relevant article, using a probabilistic approach. The second one learns the associations and then uses the results to search for linkable phrases.

Article	**Resource**	**Algorithm**	**Task**	**Data set**
Jarmasz [2003], Jarmasz and Szpakowicz [2003]	Roget	Shortest path	Word sim.	M&C, R&G, Synonyms
Mihalcea et al. [2006], corpus-based	Web / BNC	PMI-IR / LSA	Paraphrase	Microsoft
Mihalcea et al. [2006], six knowledge-based	WordNet	Shortest path, IC, etc.	=	=
Hughes and Ramage [2007]	WordNet	PPR	Word sim.	M&C, R&G, WS-353
Gabrilovich and Markovitch [2007]	Wikipedia	ESA: TF-IDF + Cosine sim.	Word sim., Doc. sim.	WS-353, Lee
Agirre and Soroa [2009]	~WordNet	PPR	WSD	Senseval-2, 3
Zesch et al. [2008]	WordNet, Wikipedia, Wiktionary	Path length, concept vectors	Word sim.	M&C, R&G, WS-353 + German
Strube and Ponzetto [2006]	Wikipedia	Shortest path, categories, text overlap	Word sim., coreference resolution	M&C, R&G, WS-353
Milne and Witten [2008a]	Wikipedia	Similarity of hyperlinks	Word sim.	M&C, R&G, WS-353
Hassan and Mihalcea [2009]	Wikipedia	Modified ESA	Cross-lingual word sim.	Translated M&C, WS-353
Syed et al. [2008]	Wikipedia	Cosine sim., spreading activation	Doc. classif.	3–100 handpicked docs
Coursey et al. [2009]	Wikipedia	PPR	Doc. classif.	150 WP articles
Ramage et al. [2009]	WordNet	PPR	Paraphrase, entailment	Microsoft, RTE
Gabrilovich and Markovitch [2009]	Wikipedia	ESA: TF-IDF + Cosine sim.	Doc. clustering	Reuters, 20NG, OHSUMED, short docs
Yeh et al. [2009]	Wikipedia	PPR	Word sim., Doc. sim.	M&C, WS-353, Lee
Present proposal	Wikipedia	Visiting Probability (*VP*)	Word sim., Doc. sim. and clustering, Classification, IR	See Sections 4.6–4.10.

Table 2.1: Comparison of the present proposal (last line) with previous work cited in this section, in terms of resources, algorithms, NLP tasks, and data sets. The abbreviations for the data sets in the rightmost column are explained in Section 4.6 . The methods are abbreviated as follows: ESA for Explicit Semantic Analysis [Gabrilovich and Markovitch, 2007, 2009], LSA for Latent Semantic Analysis [Deerwester et al., 1990], IC for Information Content [Resnik, 1995, 1999], PMI-IR pointwise mutual information using data collected by information retrieval [Turney, 2001, Mihalcea et al., 2006], and PPR for the Personalized PageRank algorithm [Haveliwala, 2003, Berkhin, 2005].

3 Symmetric Random Walk on Large Graphs: Approximation Algorithms and Similarity Learning

In this chapter we define a random walk model, Visiting Probability (*VP*), to measure proximity between two nodes in a graph. We show that using a symmetric proximity measure based on the defined random walk improves the prediction performance of the proximity measure. Moreover, we show how to apply the definition of *VP* within approximation algorithms in order to perform ranking based on *VP* for large graphs. Besides, we define community of a node according to *VP* similarity. Fast algorithms are designed to find the K-nearest neighbors and identify communities over large graphs using symmetric *VP* proximity . We demonstrate the effectiveness of the definitions and algorithms by evaluating them on a link prediction task over various networks.

In the second part of this chapter, we show that any relation between two nodes in a graph can be interpreted as the proximity between those two nodes in a latent space. Therefore, the link structure of a graph can be modeled within a similarity learning framework, in which the transformation of nodes to the latent space is trained using a discriminative model. We show how to apply this framework to learn the similarity between two nodes based on the attributes of the nodes, over large graphs. Moreover, we show that similarity learning on attributes derived from random walk scores, specifically *VP* scores here, can model and predict better the relations in the graph in comparison to learning on the network's links directly. At the end, we evaluate again the effectiveness of these models on link prediction task on various networks.

3.1 Visiting Probability

In this section, we describe our method for computing proximity between two nodes in a network. The goal is to estimate a distance between two nodes by taking into account the global connectivity of the network, and without being biased by local properties.

Indeed, the use of individual links and paths, e.g. when estimating proximity as the length of shortest path, does not take into account their relative importance with respect to the overall properties of the network, such as the number and length of all possible paths between two nodes. Moreover, the length of the shortest path is quite sensitive to spurious links. Therefore, a number of aggregated proximity measures based on random walk have been proposed in the literature, such as PageRank (including Personalized PageRank) and hitting time. Previous studied showed that these aggregated measures are more effective for link prediction rather than individual links and paths [Brand, 2005, Sarkar and Moore, 2007, Liben-Nowell and Kleinberg, 2003]. Following the same direction our proposal uses a random walk approach, and defines the proximity of two nodes as the *visiting probability (VP)*, in the long run, of a random walker going from one node to the other one. We discuss in details the differences with other popular random walk models and show how using *VP* enables us to design efficient algorithm over large graphs.

3.1.1 Notations

Let $S = \{s_i | 1 \leq i \leq n\}$ be the set of n nodes in the network. Assuming that the graph is a directed weighted multi-link graph, any two nodes s_i and s_j can be connected by one or more directed and weighted links, which can be of L different types. The structure of links of type l $(1 \leq l \leq L)$ is specified by the (sparse) matrix A_l of size $n \times n$, where $A_l(i,j)$ is the weight of the link of type l between s_i and s_j. The transition matrix C_l gives the probability of a direct (one step) transition between nodes s_i and s_j, using only links of type l. This matrix can be built from the A_l matrix as follows:

$$C_l(i,j) = \frac{A_l(i,j)}{\sum_{k=1}^{n} A_l(i,k)}$$

In the random walk process using all link types $(1 \leq l \leq L)$, let the weight w_l $(\sum_l w_l = 1)$ denote the importance of link type l. Then, the overall transition matrix C which gives the transition probability $C_{i,j}$ between any nodes s_i and s_j is $C = \sum_{l=1}^{L} w_l C_l$.

Moreover, let x_i be the normalized real-valued vector of size $1 \times M$ representing the feature vector of node s_i, where M is the number of possible attributes each node can have in the graph.

Finally, let $\vec{r}$ be the n-dimensional vector which indicates the probabilities of nodes in the network, i.e. the probability that the random walker starts from each of the nodes, so that $\sum_i \vec{r}_i = 1$.

3.1.2 Definition of Visiting Probability (*VP*)

Given the nodes s_i and s_j in the network, we compute the probability of visiting s_j for the first time when a random walker starts from s_i in the network. To compute *VP*, the following procedure provides a model of the state (node) S_t of the random walker at step t. The probability of terminating the procedure successfully, i.e. reaching s_j, is the visiting probability of s_j from s_i

which is noted VP_{ij}.

Step 0: Position the random walker on the node s_i. In other words, the initial probability vector over nodes is: $\vec{r}_i^0 = 1$.

Step t: Assuming S_{t-1} is determined, if $S_{t-1} = s_j$ then return 'success' and finish the procedure. Otherwise, with probability α, choose a value for the next node S_t according to the transition matrix C, and with probability $1-\alpha$, return 'fail'. The possibility of 'failing', or absorption probability, introduces a penalty over long paths that makes them less probable.

In the following we compute the probability of success in the above procedure which is equal to the *VP*. We introduce C' as being equal to the transition matrix C, except that in row j, $C'(j,k) = 0$ for all k. This indicates the fact that when the random walker visits s_j for the first time, it can not exit from it and its probability mass drops to zero in the next step. This modified transition matrix was defined to account for the definition of visiting probability as the probability of *first* visit of s_j (a similar idea has been proposed by Sarkar and Moore [2007]).

To compute the probability of success in the above process, we introduce the probability of success at step t, $p^t(success)$, which is $p^t(success) = \alpha^t(\vec{r}^0 C'^t)_j$. In this formula, $(\vec{r}^0 C'^t)_j$ is the j^{th} element of the vector $\vec{r}^0 C'^t$, and C'^t is the power t of matrix C'. The vector $\vec{r}^0 C'^t$ gives the probability vector at time t, noted $\vec{r}^t$.

Then, the probability of success in the process, i.e. the probability of visiting s_j starting from s_i, is the sum over all probabilities of success with different lengths:

$$p(success) = \sum_{t=0}^{\infty} p^t(success) = \sum_{t=0}^{\infty} \alpha^t(\vec{r}^0 C'^t)_j.$$

In the following sections, we introduce some properties of *VP* and afterward according to those properties we design fast algorithms to compute *VP*. But first in the following section we discuss the importance of a symmetric measure for many tasks.

3.1.3 Importance of a Symmetric Measure: Visiting Probability *'to'* vs. *'from'*

In both directed and undirected graphs VP_{ij} is not symmetric, although proximity (or similarity) should be symmetric. Moreover, we will show experimentally that a symmetric measure can model and predict the graph structure more accurately.

To compute a symmetric proximity measure between two nodes s_i and s_j, we average between their visiting probabilities VP_{ij} and VP_{ji}. A larger value of the measure indicates closer proximity. We can interpret the difference between the two scores VP_{ij} and VP_{ji} in the specific context of a given network. Let $VP_{:i}$ represent the vector of *VP* from all nodes *to* the node s_i and $VP_{i:}$ represent the vector of *VP from* s_i to all nodes in the network. If s_k is a top score node in $VP_{:i}$, then there are many high probability paths from s_k to s_i, in comparison to other nodes in the networks. Depending on the underlying meaning of each link (path) in the network, we can interpret the difference between *VP to* and *from* a node in various ways. To illustrate the difference between *VP to* and *from*, we consider an example from the network of Wikipedia articles and the hyperlinks between them. Considering the article 'Anarchism' in Wikipedia, the ten closest articles in terms of *VP from* the article, and respectively *to* it, were found to be the following ones:

- Ten closest articles according to their *VP to* the article: 'Social Revolutionary Anarchist Federation', 'James Koehnline', 'Toma Bebic', 'Social insertion' ,'Robert Graham (historian)', 'Institute for Anarchist Studies', 'Ernesto Bonomini', 'Anarchy in the Age of Dinosaurs', 'Bruno Wille'.

- Ten closest articles according to their *VP from* the article: 'Anarchism' , 'Situationist International', 'Anarchist schools of thought', 'Issues in anarchism', 'Individualist anarchism', 'Anarchism and anarcho-capitalism', 'Anarchism in Spain', 'History of anarchism', 'Anarchism and Marxism', 'Anarchism in the United States'.

The closest articles according to *VP from* an article tend to be more general topics than closest articles according to *VP to* the article. For instance, the first list above includes anarchists and anarchism communities, while the second one includes more abstract topics related to anarchism.

The behavior illustrated on this example is expected given the definition of *VP*. An article that is close *to* a target article has more paths to the target article with respect to other articles in the network, which means that the article is topically more specific with respect to the target article, as in the above example. This is caused by the definition of hyperlinks in Wikipedia: article A links to article B if B is needed to explain A, meaning typically that A is more specific (more detailed). Conversely, an article that is close *from* an initial article is likely to be a general and popular article around the topics of the initial article. If we consider the hierarchy between articles from more general to more specific ones, then articles close *to* an article are generally lower in the hierarchy, and article close *from* an article are higher. Therefore, by using the average score between *VP to* and *from* we can make a more accurate proximity (similarity) measure that can find articles at similar level in this hierarchy.

In the following section we prove some properties of *VP* that are used in the algorithms afterward.

3.1.4 Properties of *VP*

In this section, first we introduce some definitions and then we prove some propositions about the properties of *VP*. We remind first that the *VP* from s_i to s_j by using all paths with length at most t is given by VP^t_{ij} which is computed as follows:

$$VP^t_{ij} = \sum_{p=0}^{t} \alpha^p (\vec{r^0} C'^p)_j \text{ } where \text{ } \vec{r^0}_i = 1 \text{ , } C'(i,:) = \begin{cases} C(i,:) & \text{if } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

Let's denote the probability of failing the procedure to compute *VP* by using at most t steps by $lost^t_{ij}$. In other words, if we consider the traveling of random walker starting from s_i to s_j, the value of $lost^t_{ij}$ is the total probability of absorption until step t.

Moreover, we define $rem^t_{ij} = 1 - (lost^t_{ij} + VP^t_{ij})$, which is the probability of not having terminated the procedure until step t. In other words, the probability of returning neither success nor failure in the first t steps can be computed as:

$$rem^t_{ij} = \sum_{i \neq j}^{n} \alpha^t (\vec{r}^0 C'^t)_i$$

This is in fact the probability mass at time t at all nodes except s_j, the targeted node. Given these definitions we formulate the following propositions (to simplify the notations we omit the indices ij):

1. $VP^{t+1} \geq VP^t$, $lost^{t+1} \geq lost^t$, $rem^{t+1} \leq rem^t$
2. $rem^{t+1} = rem^t \times \alpha - (VP^{t+1} - VP^t)$
3. $VP \leq \underbrace{VP^t + rem^t \times \alpha}_{upper^t}$ and $upper^{t+1} \leq upper^t$

Proofs (given here in condensed form) are derived from the definition of *VP*:

1. $VP^{t+1} = VP^t + \underbrace{\alpha^{t+1} (\vec{r^0} C'^{t+1})_j}_{\geq 0} \Rightarrow VP^{t+1} \geq VP^t$

   $lost^{t+1} = \underbrace{lost^t}_{probability\ of\ failing\ until\ t} + \underbrace{(1-\alpha) \sum_{i \neq j} \alpha^t (\vec{r^0} C'^t)_i}_{probability\ of\ failing\ at\ t+1} \Rightarrow lost^{t+1} \geq lost^t$

   $rem^{t+1} = 1 - (lost^{t+1} + VP^{t+1}) \underbrace{\leq 1 - (lost^t + VP^t)}_{according\ to\ the\ above} = rem^t \Rightarrow rem^{t+1} \leq rem^t$

2. $rem^{t+1} = P(\text{'}not\ terminated\ until\ t\text{'} \wedge \text{'}not\ failed\ at\ (t+1)\text{'}) - P(\text{'}visiting\ j\ at\ (t+1)\text{'}) = rem^t \times \alpha - (VP^{t+1} - VP^t)$

3. for any $T > t$, $VP^T = VP^{T-1} + \alpha \times rem^{T-1} - rem^T = VP^{T-2} + \alpha \times rem^{T-2} + (\alpha - 1) \times rem^{T-1} - rem^T = \ldots = VP^t + rem^t \times \alpha + \underbrace{\ldots + (\alpha - 1) \times rem^{T-2} + (\alpha - 1) \times rem^{T-1} - rem^T}_{\leq 0} \leq$

$$\underbrace{VP^t + \alpha \times rem^t}_{\textit{upper bound for the value of VP}}$$

In summary, we showed that at each step t we can find the upper bound for the final value of the *VP*. Moreover, the upper bound and the value of VP^t are getting closer as t increases. We will use these propositions to design fast algorithm to perform ranking and k-nearest neighbors.

3.1.5 Relation to other Random Walk Models

In this section we discuss the relation between *VP* and two popular random walk measures which have been studied extensively and used in many applications: Hitting Time [Aldous and Fill, 2002, Liben-Nowell and Kleinberg, 2003, Sarkar and Moore, 2007, Sarkar et al., 2008] and Personalized Page Rank (PPR) or also known as random walk with restart [Haveliwala, 2003, 2002, Tong et al., 2006, Fogaras et al., 2005].

First we define with our own notations the Personalized Page Rank process started from node s_i . If $\vec{r}^{\,t}$ represents the probability distribution on all nodes in the graph at step t and $\vec{r}^{\,0}_i = 1$, then its value in personalized page rank process is given by:

$\vec{r}^{\,t} = (1 - \alpha) \times \vec{r}^{\,0} + \alpha \times \vec{r}^{\,t-1} \times C$

and consequently we define $PPR^t_{ij} = \vec{r}^{\,t}_j$ where $\vec{r}^{\,0}_i = 1$. In this context $\vec{r}^{\,0}$ is also called teleport vector because at each step with probability $1 - \alpha$ the random walker jumps to this starting distribution, $\vec{r}^{\,0}$.

In cyclic graphs, in the computation of *VP*, the loops starting from s_j and ending to s_j do not have any effect on the final score, unlike the computation of *PPR*, for which such loops boost the probability of s_j. If some nodes have this type of loops (typically “popular” nodes), then after using *PPR* they will have high probability although they might not perceived as being very close to the initial node s_i. But if the graph is acyclic, then we can show that the ranking resulting from *VP* is equal to the ranking from *PPR*. More specifically, we can show that in acyclic graphs, for every nodes s_i , s_j and s_k:

$VP_{ij} > VP_{ik} \Leftrightarrow PPR_{ij} > PPR_{ik}$

To prove the above equivalence, we expand PPR^t_{ij}, and for simplicity we denote PPR_{ij} as PPR_j:

$PPR^t_j = (1-\alpha) \times \vec{r}^{\,0}_j + \alpha(1-\alpha) \times (\vec{r}^{\,0} C)_j + \alpha^2 (1-\alpha) \times (\vec{r}^{\,0} C^2)_j + \cdots + \alpha^{t-1}(1-\alpha)(\vec{r}^{\,0} C^{t-1})_j + \alpha^t (\vec{r}^{\,0} C^t)_j$

If we consider again the definition of C' given earlier, we have $(\vec{r}^{\,0} C'^t)_j \leq (\vec{r}^{\,0} C^t)_j$, because row

j of C' is zero. But if the graph is acyclic there is no loop on j, therefore, $(\vec{r}^0 C'^t)_j = (\vec{r}^0 C^t)_j$.

$$\begin{aligned} PPR^t_j &= (1-\alpha) \times (\underbrace{\vec{r}^0_j}_{0} + \underbrace{\alpha(\vec{r}^0 C')_j + \alpha^2(\vec{r}^0 C'^2)_j + \cdots + \alpha^{t-1}(\vec{r}^0 C'^{t-1})_j}_{VP^{t-1}_j}) + \alpha^t(\vec{r}^0 C'^t)_j \\ &= \underbrace{VP^{t-1}_j + \alpha^t(\vec{r}^0 C'^t)_j}_{VP^t_j} - \alpha \times VP^{t-1}_j = VP^t_j - \alpha \times VP^{t-1}_j \end{aligned} \tag{3.1}$$

We use the fact that both *VP* and *PPR* converge when t increases, therefore:

$PPR_j = (1-\alpha) \times VP_j \Rightarrow VP_{ij} > VP_{ik} \Leftrightarrow PPR_{ij} > PPR_{ik}$

If the graph is cyclic, then the *PPR* score of nodes with self loops is boosted more than the value they really deserve. Usually popular nodes have these kinds of self loops and may inaccurately appear between top score nodes.

Moreover, as we explained earlier, for many applications a symmetric measure can represent more accurately the proximity between nodes. The definition of *VP* – as the probability of visiting the target node for the first time – allows us to define a symmetric proximity measure straightforwardly, and more importantly to design fast algorithms to apply this symmetric measure to large networks, as we discuss in the next section (3.2).

To emphasize again the importance of a symmetric measure for many applications, we illustrate our position with the results of the following experiment. Given a query node in a network, we exclude one of its neighbors and use random walk measures to rank other nodes by their proximity, in order to find the missing neighbor. Table 3.1 shows the recall at 50 averaged over 1000 randomly chosen nodes as query nodes on two networks, Amazon products networks (nodes: 548,551, edges: 1,231,439) and Wikipedia articles network (nodes: 1,264,611, edges: 35,214,537). Amazon products networks is described in Leskovec et al. [2007], consisting of products in Amazon online shop and the links between them comes directly from the Amazon website: for each product, some of the co-purchased products are shown by Amazon. Each product in the network is connected to the products which are claimed to be mostly co-purchased by the Amazon website. Wikipedia network is consisting of English Wikipedia articles and the hyperlinks between them.

Random Walk Model	R@50 Amazon Products	R@50 Wikipedia Articles
PPR	66.10	39.40
Symmetric *VP*	79.80	42.80

Table 3.1: Average recall at 50 (in percentage) for 1000 randomly chosen nodes on two networks: Amazon Products and Wikipedia articles

The experimental results on these two large networks show that the symmetric *VP* can represent a more accurate proximity measure for link formation in comparison to *PPR*.

A second popular measure is hitting time. The hitting time from s_i to s_j is defined as the average number of steps a random walker would take to visit s_j for the first time in the graph. If we use the same notations, the hitting time from s_i to s_j is $H_{ij} = \sum_{t=0}^{\infty} t(\vec{r}^0 C'^t)_j$. Hitting time is more sensitive to long paths in comparison to *VP* (t in comparison with α^t in the formula), which might introduce more noise, while *VP* reduces the effect of long paths sooner in the walk.

Moreover, because of the linear nature of the penalty for long paths (t in the above formula), it is not possible to determine an upper bound (or lower bound) on H scores and therefore, designing fast algorithms to perform various tasks is not simple. On the contrary, in the definition of *VP*, the penalty for long paths is increasing exponentially (α^t in the above formula) and therefore, we can determine upper bounds on *VP* scores and use them for designing fast algorithms, as we describe in the next section.

In previous studies, truncated hitting time was used to overcome these problems [Sarkar and Moore, 2007, Sarkar et al., 2008], i.e. only paths of length at most T were used in the computation, and any path with length more than T was assumed to have the length T. Although this hard threshold approach solves the above problems heuristically, it still leaves open a difficult design choice of T – with performance being very sensitive to its value. On the contrary, α^t represents a soft threshold which leads to higher performance and is less sensitive to the choice of parameter. Table 3.2 shows the average recall at 10 for 1000 query nodes on two networks: Facebook of Carnegie Mellon University (nodes: 6637, edges: 249967) and Wikipedia articles (nodes: 1,264,611, edges:35,214,537). The Facebook data set is described in Traud et al. [2011], and is consisting of students and faculty members of Carnegie Mellon University and the friendship between them. The parameters of the algorithms are tuned on a validation set ($T = 3$, $\alpha = 0.4$, $\varepsilon = 0.1$).

Random Walk Model	R@10 Facebook	R@10 Wikipedia Articles
Hitting Time	24.70	20.80
VP	25.80	22.60

Table 3.2: Average recall at 10 (in percentage) for 1000 randomly chosen nodes on two networks: Facebook and Wikipedia articles

3.2 Approximations: ε-Truncated Visiting Probability

The above definition of the random walk procedure has one direct consequence: computation can be done iteratively and can be truncated after T steps when needed, while the error upper bound remains guaranteed. This helps especially to maintain computation time within acceptable limits. Truncation makes sense as higher order terms (for longer paths) get smaller with larger values of t because of the α^t factor. Moreover, besides making computation tractable, truncation reduces the effect of longer (hence less reliable) paths on the computed value of $p(success)$. Indeed, *VP* to popular vertices (i.e. to which many links point) might be quite high even when they are not perceived as being close to the starting node, due to the high number of long paths toward them, and truncation conveniently reduces the importance of such vertices. We propose in this section two methods for truncating *VP*.

3.2.1 Path Insensitive ε-Truncated Visiting Probability

The simplest method, called path insensitive ε-truncated *VP*, truncates the computation for all the paths when the error is known to be smaller than ε. To compute the upper bound on the number of iterations needed before truncation, we make use of the propositions we introduced in the last section: $\varepsilon_t = VP_{ij} - VP^t_{ij} \leq rem^t_{ij} \times \alpha = \sum_{i \neq j}^n \alpha^{t+1}(\vec{r}C'^t)_i$.

So, if VP^t is used as an approximation for *VP*, then an upper bound for this approximation error ε_t is the right term of the above inequality. This term decreases over time because α^{t+1} and $\sum_{i \neq j}^n (rC'^t)_i$ are both decreasing over time, therefore ε_t decreases when t increases and its computation is truncated when the error is acceptable. In the rest of this document we refer to this method as *truncated visiting probability*.

3.2.2 Path Sensitive ε-Truncated Visiting Probability

A second approach, referred to as path sensitive ε-truncated *VP*, truncates paths with lower probabilities in earlier steps and lets paths with higher probabilities continue more steps. Given that the probability of being at s_i at time step t is $\alpha^t(rC'^t)_i$, if this is neglected and set to zero, then the error caused by this approximation is at most $\alpha^t(rC'^t)_i$. Setting this term to zero means exactly that paths that are at s_i at time step t are no longer followed afterwards. So, in ε-truncation, paths with a probability smaller than ε are not followed, i.e. when $\alpha^t(rC'^t)_i \leq \varepsilon$. This approach is faster to compute than the previous one, but no upper bound of the error could be established. We use the path sensitive ε-truncated *VP* in some of our experiments in the next chapter, leading to competitive results in an acceptable computation time.

3.2.3 Truncated Visiting Probability Between all Vertices and a Query

In many applications – including clustering, information retrieval and recommendation systems – given a query node the rest of nodes should be sorted based on a proximity measure. In this section we design algorithms to compute truncated *VP* between a query node (vertex) and the rest of the nodes (vertices) in a large graph within an acceptable time limit. We explained earlier that there is a difference between *VP from* and *to* a vertex. In the following we design algorithms for both *VP* from a query node *to* the rest of nodes, and the reverse, i.e. *from* all nodes to a query node.

Truncated Visiting Probability from all Vertices to a Vertex

It is possible to compute *VP* from all nodes towards s_j at the same time, using the following recursive procedure to compute truncated *VP*. This procedure follows from the recursive definition of *VP* given in Section 3.1.2, which states that $VP_{ij} = \alpha \times \sum_k C'(i,k)\, VP_{kj}$. Therefore, using dynamic programming, it is possible to compute truncated *VP* from all nodes to s_j in $O(ET)$ steps, where E is the number of edges of the network and T is the maximum number of iterations that will be performed, which is at most $\log_\alpha \varepsilon$. Algorithm 1 shows the pseudo code for the described method.

Algorithm 1 Truncated *VP* from all nodes to s_j : VPto(α,C$'$,ε,j)

```
$VP_i^0 = 0\ \forall i \neq j$ , $VP_j^0 = 1$
$rem_i^0 = 1\ \forall i \neq j$ , $rem_j^0 = 0$
$error_i = 1\ \forall i \neq j$ , $error_j = 0$
$T_i = 1$
while $\exists i$, $error_i \geq \varepsilon$ do
    for $\forall i$, $error_i \geq \varepsilon$ do
        $VP_i^{T_i} = \alpha \times \sum_k C'(i,k)\, VP_k^{T_i - 1}$
        $rem_i^{T_i} = \alpha \times rem_i^{T_i-1} - (VP_i^{T_i} - VP_i^{T_i-1})$
        $error_i = \alpha \times rem_i^{T_i}$
        $T_i = T_i + 1$
    end for
end while
```

Truncated Visiting Probability from a Vertex to all Vertices

To compute truncated *VP* from s_i to all nodes in the network, we should distinguish two cases, cyclic and acyclic graphs. If the graph is acyclic we can design a dynamic programing algorithm, similar to the algorithm above, that compute *VP* to all nodes efficiently. *VP* considers the first visit of the target node, therefore, when the graph is cyclic we should keep track of all nodes that have been visited to avoid the effect of loops. In this case, the algorithm has more complexity as it stores all the paths expanded from the node s_i. In the following we first introduce the algorithm for acyclic graphs, then we design the algorithm for cyclic graphs, and eventually we introduce a sampling approximation algorithm to approximate truncated *VP* from a node to all nodes in dense large cyclic graphs.

Truncated Visiting Probability from a Vertex to all Vertices: Acyclic

If the graph is acyclic we do not need to worry about loops and, therefore, it is possible to compute *VP* to all nodes from s_i at the same time, using again a recursive procedure. It is possible to define *VP* with the following recursive definition: $VP_{ij} = \alpha \times \sum_k VP_{ik} \times C'(k,j)$. Therefore, using dynamic programming, it is possible to compute truncated *VP* to all nodes from s_i in $O(ET)$ steps, where E is the number of edges of the network and T is the maximum number of iterations that will be performed, which is at most $\log_{\alpha} \varepsilon$. Algorithm 2 shows the pseudo code for the described method.

Algorithm 2 Truncated *VP* to all nodes from s_i : VPfromAcyc(α, C$'$, ε, i)

```
$VP_j^0 = 0 \; \forall j \neq i \;,\; VP_i^0 = 1$
$rem_j^0 = 1 \; \forall j \neq i \;,\; rem_i^0 = 0$
$error_j = 1 \; \forall j \neq i \;,\; error_i = 0$
$T_j = 1$
while $\exists j,\ error_j \geq \varepsilon$ do
  for $\forall j,\ error_j \geq \varepsilon$ do
    $VP_j^{T_j} = \alpha \times \sum_k VP_k^{T_j - 1} \times C'(k, j)$
    $rem_j^{T_j} = \alpha \times rem^{T_j - 1} - (VP_j^{T_j} - VP_j^{T_j - 1})$
    $error_j = \alpha \times rem_j^{T_j}$
    $T_j = T_j + 1$
  end for
end while
```

Truncated Visiting Probability from a Vertex to all Vertices: Cyclic

If the graph is cyclic we cannot use the previous algorithm as it is ignorant about the loops. To compute *VP* to all nodes from s_i we start from s_i, and expand paths originating from s_i until the error of *VP* to all nodes is smaller than the ε.

In a nutshell, the main idea of the algorithm is the following one. At iteration t we expand the path with maximum probability one step forward to find all paths generated from this path. Consequently we compute the probability of each new path based on the probability of the originating path and the transition probability from C. If the last vertex of a new path has not been previously visited in that path, then the *VP* to that vertex is increased by the probability of this path. The iterations go on until the error becomes smaller than ε. Algorithm 3 shows the pseudo code for this algorithm.

If we consider that the average number of edges per node is b, then there are b^T paths of length T in average. The worst time complexity of the algorithm is linear with the number of paths at t. Therefore, if the graph is large and dense the above algorithm is not applicable because it is too time-consuming. We will thus propose an approximation algorithm for truncated *VP* to all nodes based on sampling.

Approximated Truncated Visiting Probability from a Vertex to all Vertices: Sampling

As stated above, if the graph is large and dense, then the previous algorithm to compute *VP* from s_i to the other nodes is too time-consuming. To obtain an error bound of ε, only paths of length at most $T = \lceil \log_{\alpha} \varepsilon \rceil$ should be considered. We design a sampling algorithm which samples paths of length at most T, originating from s_i, to approximate the value of truncated

Algorithm 3 Truncated *VP* to all nodes from s_i : VPfrom(α, C$'$, ε, i)

```
$VP_j = 0 \; \forall j \neq i$ , $VP_i = 1$
$rem_j = 1 \; \forall j \neq i$ , $rem_i = 0$
$error_j = 1 \; \forall j \neq i$ , $error_i = 0$
$Paths.add(i)$
$Pr.add(1)$
while $\exists j$, $error_j \geq \varepsilon$ do
    $p = Paths.maxProb()$
    $j = lastNode(p)$
    $Children = getChildren(j)$
    $rem = rem - (1-\alpha) \times Pr(p)$
    for $\forall c \in Children$ do
        $Paths.add(p\text{->}c)$
        $Prob = \alpha \times Pr(p) \times C'(j,c)$
        $Pr.add(Prob)$
        if $c \notin p$ then
            $VP_c = VP_c + Prob$
            $rem_c = rem_c - Prob$
        end if
    end for
    $Paths.remove(p)$ , $Pr.remove(p)$
    $error = \alpha \times rem$
end while
```

VP from s_i to all other nodes.

The sampling involves running M independent T-length random walks from s_i. To approximate VP^T to node s_j from s_i, if s_j has been visited for the first time at $\{t_{k_1}, \cdots, t_{k_m}\}$ time steps in the M samples, then the T-truncated *VP* to s_j can be approximated by the following average: $\hat{VP^T}_{ij} = (\sum_l \alpha^{t_{k_l}})/M$. The algorithm 4 shows the pseudo code for this sampling algorithm.

According to the proposed method, it is possible to approximate truncated *VP* from s_i to all nodes in the network in $O(MT)$ time, where M is number of samples. It remains to find out how many samples should be used to obtain the desired approximation level, a question that is answered by the following theorem.

Theorem 1. *For any vertex, the estimated truncated VP approximates the exact truncated VP^T to that vertex within ε with a probability larger than $1-\delta$ if the number of samples M is larger than $\frac{\alpha^2 \ln(2n/\delta)}{2\varepsilon^2}$.*

Proof. The proof of this theorem, inspired by the proof in Sarkar et al. [2008], is given below.

Let us note the estimation of a variable X by $\hat{X}$, and suppose that node s_j has been visited for the first time at $\{t_{k_1}, \cdots, t_{k_M}\}$ time steps in the M samples. We define the random variable X^l by $\alpha^{t_{k_l}}/M$, where t_{k_l} indicates the time step at which s_j was visited for the first time in lth

Algorithm 4 Truncated *VP* to all nodes from s_i : VPfromSample(α, C′, T, M, i)

```
$VP_j = 0\ \forall j \neq i$ , $VP_i = 1$
**for** $s = 1 : M$ **do**
  $samples.add(i)$
  **for** $t = 1 : T$ **do**
    $CurrentNode = samples.getLast$
    $c$ = choose randomly one children of $CurrentNode$ according to $C'$
    **if** $c \notin samples$ **then**
      $VP_c += \alpha^t$
    **end if**
  **end for**
  $samples.clear$
**end for**
$VP_j = \frac{VP_j}{M}\ \forall j \neq i$
```

sampling. If s_j was not visited at all, then $X^l = 0$ by convention. The l random variables X^l ($k_1 \leq l \leq k_M$) are independent and bounded by 0 and 1 ($0 \leq X^l \leq 1$). We have $\hat{VP^T}_{ij} = \sum_l X^l = (\sum_l \alpha^{t_{k_l}})/M$ and $E(\hat{VP^T}_{ij}) = VP^T_{ij}$. So, by applying Hoeffding's inequality, we have:

$$P(|\hat{VP^T}_{ij} - E(\hat{VP^T}_{ij})| \geq \varepsilon) \leq 2exp(-\frac{2M\varepsilon^2}{\alpha^2})$$

If the probability of error must be at most δ, then setting the right side to a value smaller than δ gives the bound for M that is stated in our theorem.

As a consequence, we have the following lower bound for M if we look for an ε-approximation for all possible s_j with probability at least $1-\delta$. We use the union bound and Hoeffding's inequality to prove that:

$$P(\exists j \in \{1 \cdots n\}, |\hat{VP^T}_{ij} - E(\hat{VP^T}_{ij})| \geq \varepsilon) \leq 2n \times exp(-\frac{2M\varepsilon^2}{\alpha^2})$$

which gives the desired lower bound $M \geq \frac{\alpha^2 \ln(2n/\delta)}{2\varepsilon^2}$.

□

3.3 Fast Computing of Nearest Neighbors of a Query Node

In many applications including recommendation, retrieval and KNN classification, given a query node only the few closest nodes to the query are important. It is theoretically possible to directly use the algorithms proposed in the previous section to find the truncated *VP* between a given (query) node and all other nodes, and then simply return the K closest nodes after sorting all nodes. However, in most graphs, it is possible to design even faster algorithm to find the closest nodes to a given query node in terms of *VP* similarity.

3.3.1 Fast K-Nearest Neighbors of a Query Node

We can make use of the propositions we know about *VP* scores to design a fast K-nearest neighbors algorithm. We showed that for each t we can determine an upper bound for the value of *VP*, and moreover, we showed that the upper bound is converging towards VP^t when t increases. Using these facts, the fast algorithm is intuitively as follows.

Given a query node, at the beginning we have no information about *VP* between the query node and any other node in the network. In the next step, we consider all paths of length 1 *from* and *to* the query node. After that, we have the VP^1 and upper bound for the value of *VP* for all nodes in the graph. Still, the difference between the upper bound and VP^1 might be large for many nodes at this step. We sort the nodes by their decreasing VP^1 values and pick the node at position K.

Two scenarios might happen at this stage. First, it may happen that there is no node at any position after K that has an upper bound larger than the VP^1 of the node at position K. In this case we return the first K nodes and terminate the algorithm, because no node at position after K can have a larger *VP* than any node at any position below K, even if we develop paths of all lengths.

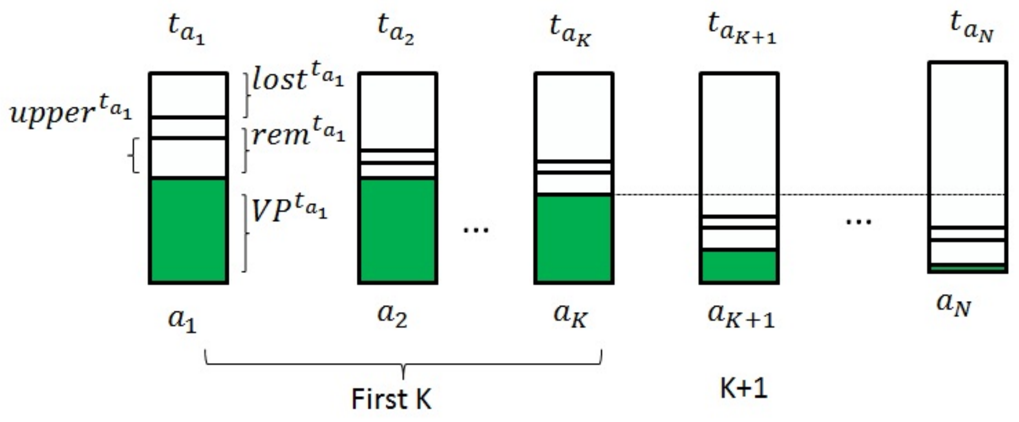


Figure 3.1: Fast KNN algorithm: schematic representation of the situation in which the VP^t of the node at the position K is larger than the upper bound of all the nodes after it. The notations are those from Algorithm 5.

Second, there might be nodes at positions after K that have upper bounds larger than VP^1 of the node at position K. In this case, we compute the VP^2 for these nodes and the node at position K and sort again the nodes based on these new values. Again, we consider the node at position K (which might have of course changed after sorting) and repeat the same algorithm, which will either terminate and return the first K nodes, or will consider again longer paths. In the long run, the recursion will certainly terminate because the upper bounds converge to VP^t eventually. Figure 3.1 shows schematically the situation in which the VP^t of the node at the position K is bigger than the upper bound of the nodes ranked after it.

In the algorithm 5 we give the pseudo code of the algorithm. We will analyze the efficiency of this algorithm experimentally later in this chapter.

Algorithm 5 KNN for s_i : KNN(α, C$'$, ε, i, K)

```
$VP^0_j = 0 \;\forall j \neq i$ , $VP^0_i = 1$
$rem^0_j = 1 \;\forall j \neq i$ , $rem^0_i = 0$
$error_j = 1 \;\forall j \neq i$ , $error_i = 0$
$upper_j = VP^0_j + \alpha \times rem^0_j$
$T_j = 1$
$sorted = sort(VP^T)$
while $(\exists j \in sorted(K+1:N)\,, VP^T_{sorted(K)} \leq upper^{T_j}_j) \wedge (error_j \geq \varepsilon)$ do
    for $\forall j \in sorted(K:N)\,, VP^T_{sorted(K)} \leq upper^{T_j}_j) \wedge (error_j \geq \varepsilon)$ do
        $VP^{T_j}_j$ = compute based on $VP^{T_j-1}$
        $rem^{T_j}_j = \alpha \times rem^{T_j-1} - (VP^{T_j}_j - VP^{T_j-1}_j)$
        $error_j = \alpha \times rem^{T_j}_j$
        $upper_j = VP^{T_j}_j + \alpha \times rem^{T_j}_j$
        $T_j = T_j + 1$
    end for
    $sorted = resort(VP^T)$
end while
```

3.3.2 Community of the Query Node

In this section, we consider small-world networks and briefly survey the fact that many networks have been shown to be small-world networks in previous studies. Then, we show that there is a well-defined community for a query node in a small-world network, and design a fast algorithm based on VP to perform community identification for a given query node.

A small-world network is a type of graph in which most nodes are not neighbors of one another, but most nodes can be reached from any other node by a small number of transitions [Watts and Strogatz, 1998]. In other words, nodes form "local" communities and these communities are connected together by short paths.

It has been shown that many real-world complex systems can be modeled by small-world networks, such as for instance metabolite processing networks [Wagner and Fell, 2001], chemical-reaction networks [Alon et al., 1999], neural networks [Watts, 2003], food webs [Pimm et al., 1991], social networks [Carrington et al., 2005], scientific-collaboration networks [Vanraan, 1990] and the entire World Wide Web [Adamic and Huberman, 2000, Barabási and Albert, 1999].

Therefore, if we consider a random walker starting from a query node in a small-world network, only few nodes are likely to be visited in this process, though not necessarily the direct neighbors of the query node. To justify experimentally this statement, let us consider the following process. Given a network and a query node, we sort all the nodes in the network according to their symmetric *VP* scores. To increase the robustness of our experiment, we repeat this process for a large number of query nodes and average for each query node the scores of the nodes in the *VP* ranking list. Moreover, before averaging, we remove the direct neighbors of the query node from this ranked list. The final descending ranked list gives us the average *VP* scores of each position considering only the nodes that are not directly linked. For example, the first number is the average *VP* score of the closest (in terms of *VP*) not-linked node to the query node.

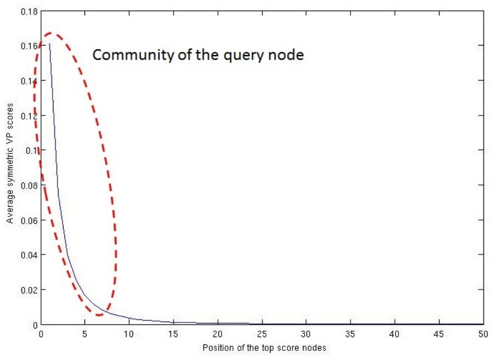
Figure 3.2 shows the average *VP* scores at the top 50 nodes in the ranked list explained above for the five following networks: Cora (average on all nodes), CiteSeer (average on all nodes), WebKB (average on all nodes), Amazon Products co-purchased network (average on 1000 nodes chosen randomly), English Wikipedia network (average on 1000 nodes chosen randomly). The detailed description of the data sets WebKB, CiteSeer and Cora can be found in Section 3.3.3. It appears that for any query node, there are only very few nodes (3–5) with high *VP*, as the curves decrease rapidly. The remaining of nodes are not likely to be visited by a random walker starting from the query node. Intuitively, by looking at the figures, we can find the size of the community of a query node, and we formalize below this intuitive concept of community.

If the nodes are sorted according to their *VP* as in Figure 3.2, then we can intuitively conclude that the community boundary is where the *VP* scores of nodes become negligible. If we note the ranked list of *VP* values between s_i and the rest of (not directly connected) nodes as $VP_{a_1}, VP_{a_2}, \ldots, VP_{a_N}$, then the community of the node s_i, noted S_i, can be defined as:

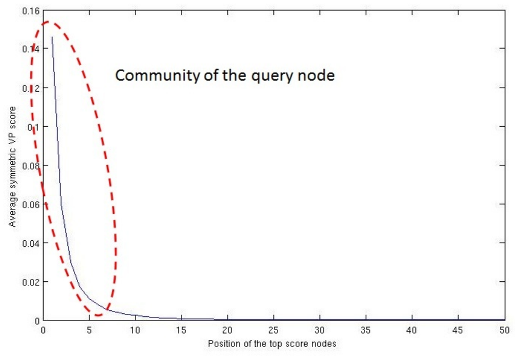
$$S_i = \{s_{a_1}, s_{a_2}, \ldots s_{a_j}\} \ \textit{so that}\ \frac{\sum_{h=1}^{j} VP_{a_h}}{\sum_{h=1}^{N} VP_{a_h}} \geq Tr, \text{ where } Tr \text{ is a fixed threshold.}$$

Fast Community Identification

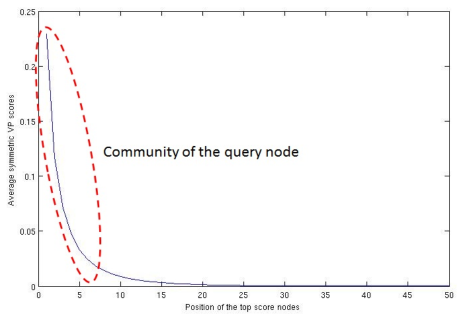
We propose now a fast algorithm to identify the community of a given query node. Similar to the fast K-nearest neighbors algorithm we proposed in Section 3.3.1, we design an algorithm that finds the community of a node without expanding all paths *from* and *to* the query node. Using the previous propositions about *VP*, we can show that $\frac{\sum_{h=1}^{j} VP^t_{a_h}}{\sum_{h=1}^{N} upper^t_{a_h}} \leq \frac{\sum_{h=1}^{j} VP_{a_h}}{\sum_{h=1}^{N} VP_{a_h}}$ for any given t, because always $upper^t_j$ is bigger than VP_j. Therefore, if at any position j in the ranked list we have $\frac{\sum_{h=1}^{j} VP^t_{a_h}}{\sum_{h=1}^{N} upper^t_{a_h}} \geq Tr$, then we know the community of s_i is until j.



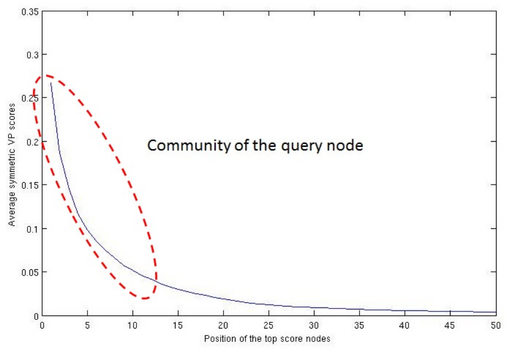
(a) Cora Network



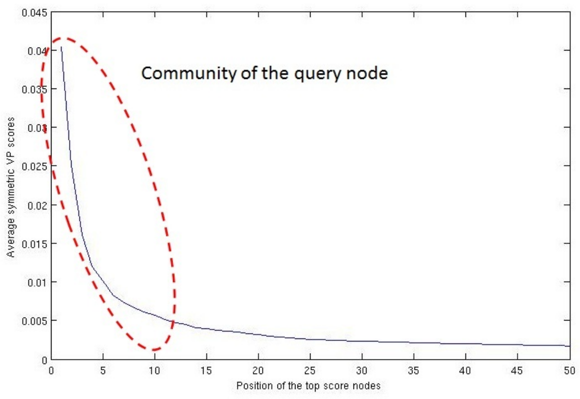
(b) CiteSeer Network



(c) WebKB Network



(d) Amazon Network



(e) Wikipedia Network

Figure 3.2: Average *VP* for the top 50 not-linked closest nodes on Cora, CiteSeer, WebKB, Amazon Products and Wikipedia networks. The *VP* values decrease rapidly after the first few nodes.

The algorithm in summary is the following. At the first step we consider only paths of length one *to* and *from* the query node. All the nodes are sorted according to their VP^1. If there exist a position $j \le K$ that satisfies the conditions above, the community is until j (note that K is the maximum size of the community that must be given by the user). If there is no such a position j, then we expand all paths by one and consider all paths of length at most 2, and sort again the nodes according to their new *VP* scores. Again, if there exists a position that satisfies the condition, the algorithm is terminated; otherwise, we expand all the paths by one more node. Algorithm 6 gives the pseudo code of the explained algorithm.

Algorithm 6 Community Identification for s_i : Comm(α, C$'$, ε, i, K)

```
  $VP_j^0 = 0\ \forall j \neq i$ , $VP_i^0 = 1$
  $rem_j^0 = 1\ \forall j \neq i$ , $rem_i^0 = 0$
  $error_j = 1\ \forall j \neq i$ , $error_i = 0$
  $upper_j = VP_j^0 + \alpha \times rem_j^0$
  $T = 1$
  $sorted = sort(VP^T)$
  while ($\exists j, error_j \geq \varepsilon$) do
    for $j = 2 : K$ do
      if $\frac{\sum_{h=1}^{j} sorted_h}{\sum_{h=1}^{K} upper_h} \geq Tr$ then
        return $sorted(1:j)$
      end if
    end for
    $T = T + 1$
    for $\forall j \neq i$ do
      $VP_j^T$ = compute based on $VP^{T-1}$
      $rem_j^T = \alpha \times rem_j^{T-1} - (VP_j^T - VP_j^{T-1})$
      $error_j = \alpha \times rem_j^T$
      $upper_j = VP_j^T + \alpha \times rem_j^T$
    end for
    $sorted = resort(VP^T)$
  end while
  for $j = 2 : K$ do
    if $\frac{\sum_{h=1}^{j} sorted_h}{\sum_{h=1}^{K} sorted_h} \geq Tr$ then
      return $sorted(1:j)$
    end if
  end for
```

3.3.3 Experimental Results on Link Prediction

In this section, we perform experiments on various networks to validates the above assumptions empirically. We use three data sets which have been frequently used for the collective *classification* problem (more details about them can be found in Sen et al. [2008]):

- The *Cora* dataset consists of papers in the field of machine learning. The papers were selected so that in the final corpus every paper cites or is cited by at least one other paper.

- The *CiteSeer* dataset contains a selection of papers in the field of computer science. Again, the papers were selected so that in the final corpus every paper cites or is cited by at least one other paper.

- The *WebKB* dataset contains webpages and hyperlinks between them, gathered from four different universities.

A set of binary features or attributes is defined for each object (in all data sets), which are the presence or absence of a given word. All data sets are in the form of directed graphs. Statistics for the data sets are given in Table 3.3.

Data Set	**Edges**	**Attributes**
Cora	5429	1440
CiteSeer	4715	3709
WebKB	1608	1709

Table 3.3: Number of edges and attributes of the data sets.

We test our proximity measures over a link prediction task by using a ranking framework, as follows. Link prediction can be formulated as a ranking problem: given a query node, all other nodes must be sorted according to the likelihood of creating a link between them and the query node. In our proposal, the nodes are ranked using the *VP* proximity measure, in comparison to other measures proposed in the literature. We perform ten-fold cross validation on links and report the average precision and recall. In other words, we divide the set of edges to ten random sets and test the link prediction performance of one set when all other sets are available. The experiments are done with $\alpha = 0.6$.

Tables 3.4, 3.5 and 3.6 show the performance of different methods on the networks. The Adamic & Adar proximity measure, which is based on common neighborhood, showed very high performance in previous studies [Adamic and Adar, 2001, Liben-Nowell and Kleinberg, 2003] and therefore, we used it here as a competitive baseline. We can observe that, over all data sets, exploring more than the direct neighbors by using random walk models improves the performance in comparison to Adamic & Adar proximity.

Moreover, in all tested networks, the symmetric measure gives us the highest performance at 5 in comparison to other random walk models, which confirms again the effectiveness of the proposed symmetric measure. Tables 3.4, 3.5 and 3.6 also report the average of the longest paths that are needed to be explored by *VP* in order to find the closest 5 nodes to each query node: the longest path we need to expand is less than 9 steps in all three networks.

Finally, we observe that the highest precision and recall are reached by the method that identifies the community of each query node presented in Section 3.3.2 above. The community identification algorithm can successfully find the nodes which are likely to form a link with the query node and outperforms the methods based on ranking of the nodes.

Model	Precision	Recall	Avg. longest path	Community size
Adamic & Adar @5	3.18	12.40	-	-
PPR @5	4.06	15.84	-	-
VP @5	4.25	16.32	8.8	-
VP Sym. @5	**5.17**	**20.66**	8.86	-
VP + Comm. Identification	8.03	19.66	8.96	6.26
VP Sym.+ Comm. Identification	**8.29**	**23.97**	8.89	6.72

Table 3.4: Average precision and recall on the Cora network for the link prediction task described in Section 3.3.3

Model	Precision	Recall	Avg. longest path	Community size
Adamic & Adar @5	2.13	8.93	-	-
PPR @5	2.65	10.89	-	-
VP @5	2.83	11.46	8.93	-
VP Sym. @5	**3.10**	**12.96**	8.89	-
VP + Comm. Identification	**6.54**	13.06	8.97	3.24
VP Sym.+ Comm. Identification	6.33	**14.73**	8.92	3.70

Table 3.5: Average precision and recall on the CiteSeer network for the link prediction task described in Section 3.3.3

Model	Precision	Recall	Avg. longest path	Community size
Adamic & Adar @5	2.41	10.14	-	-
PPR @5	3.78	15.41	-	-
VP @5	3.80	14.88	8.85	-
VP Sym. @5	**7.26**	**31.56**	8.68	-
VP + Comm. Identification	5.50	17.19	8.92	3.87
VP Sym.+ Comm. Identification	**10.94**	**33.75**	8.63	4.88

Table 3.6: Average precision and recall on the WEBKB network for the link prediction task described in Section 3.3.3

3.4 Learning Embeddings for Latent Space Model of Networks

Homophily – the fact that two nodes having similar ”social characteristics” have a higher probability to be linked – is an important and widely accepted assumption in the study of social networks. The homophily principle applies to network relations of many types, such as marriage, friendship, work, advice, support, information transfer and co-membership

[McPherson et al., 2001]. The "social characteristics" of nodes are derived from various attributes of a node, for example, age, sex, geographic location, college/university, work place, hobbies/interests. Homophily, though, might not hold in the space of the attributes: for example, marriage only occurs between different genders (in most countries).

The space that is formed by the "social characteristics" or "latent features" is called social space or latent space. It is assumed that homophily holds in this latent space. Statistical models have been previously designed to associate every node with locations in this latent space. Links are more likely if the entities are close in the latent space. In other words, all the pairwise links are independent, conditioned on their latent positions, i.e. distances in the latent space [Hoff et al., 2001].

We introduce here a large-margin discriminative latent space learning algorithm to model (social) networks by using the nodes' attributes. Intuitively, we will learn how to transform and place the nodes in the latent space according to their attributes – learning a linear transformation from the attribute's space to the latent space. In the following, we first explain the model, and then we show how we can apply it so that we can project random walk scores into the latent space.

3.4.1 Learning Embeddings

We remind from Section 3.1.1 that x_i is the normalized real-valued vector of size $1 \times M$ representing the feature vector of node s_i, where M is the number of possible attributes each node can have. We note $A_{M\times D}$ the matrix that defines the linear transformation of each node s_i to the latent space of dimension D. Therefore, $x_i A$ is the point representing s_i in the latent space.

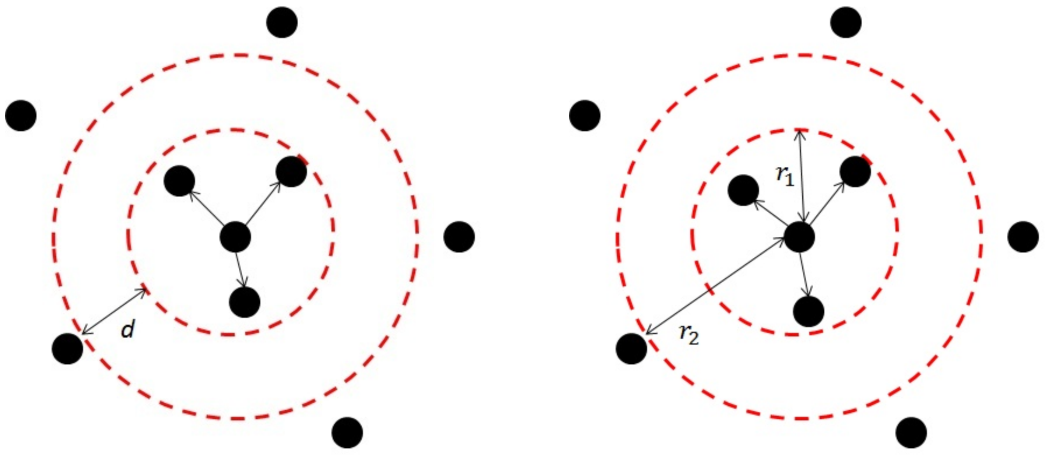
To train the parameters of the model (i.e. A), we assume that the homophily principle holds in the latent space. Therefore, the objects that are connected in the network should be closer together, in the latent space, than non-connected ones. To enforce this principle, we design and optimize two different objective functions, which both consist of a hinge loss function. In the following, we briefly explain both objective functions.

Model 1: Margin between connected and non-connected objects

To apply the homophily principle, we define an objective function so that connected objects are closer than the non-connected ones with a margin d. Figure 3.3(a) shows this objective function schematically. The corresponding node for node s_i in the latent space is $x_i A$, therefore, the similarity between s_i and s_j in the latent space is $x_i AA' x'_j$, the dot product between the transformed nodes. In directed networks, we need an asymmetric similarity function, which we can build by using two transformations A and B, so that similarity from s_i to s_j is

$x_i AB' x'_j$. Putting all together the objective function we minimize is the following:

$$Min\ L = \sum_{(i,j)\in E\ ,\ (i,z)\notin E} max(0, d - x_i AB' x'_j + x_i AB' x'_z) \tag{3.2}$$



(a) Distance between connected objects and non-connected objects should be larger than a margin d

(b) Connected objects should be closer than r_1 and non-connected objects should not be closer than r_2

Figure 3.3: Two latent space models

Model 2: Connected objects closer than r_1 and non-connected objects not closer than r_2

According to this second objective function connected nodes should be closer than r_1 and non-connected objects should not be closer than r_2. Figure 3.3(b) shows this objective function schematically. The second objective function is more general and encompasses the first one, and therefore, it is harder to optimize.

$$Min\ L = \sum_{(i,j)\in E} max(0, r_1 - x_i AB' x'_j) + \sum_{(i,j)\notin E} max(0, x_i AB' x'_z - r_2) \tag{3.3}$$

The summations in the above objective functions are not tractable to optimize if the graph is large, which is the case in our work. To overcome this problem, we make use of a stochastic gradient descent algorithm in which we optimize the objective function iteratively. For the first model, at each iteration we choose randomly one triple of nodes (i, j, z) and optimize by gradient descent on this triple only. For the second model, we choose randomly one $(i, j) \in E$ and optimize the objective function by gradient descent on (i, j), then we choose randomly the pair $(i, z) \notin E$ and optimize the objective function accordingly.

The gradients of the objective functions at step t are shown below, for the first model:

$$\bigtriangledown L_t(A) = \{ \begin{array}{rl} x_i'(x_z B - x_j B) & \text{if } x_i AB'x_j' - x_i AB'x_z' < d \\ 0 & \text{otherwise} \end{array}$$

$$\bigtriangledown L_t(B) = \{ \begin{array}{rl} (x_z - x_j)' x_i A & \text{if } x_i AB'x_j' - x_i AB'x_z' < d \\ 0 & \text{otherwise} \end{array}$$

and the second model:

$$\bigtriangledown L_t(A) = \{ \begin{array}{rl} -x_i'(x_j B) & \text{if } (i,j) \in E \text{ , } x_i AB'x_j' < r_1 \\ x_i'(x_j B) & \text{if } (i,j) \notin E \text{ , } x_i AB'x_j' > r_2 \\ 0 & \text{otherwise} \end{array}$$

$$\bigtriangledown L_t(B) = \{ \begin{array}{rl} -x_j'(x_i A) & \text{if } (i,j) \in E \text{ , } x_i AB'x_j' < r_1 \\ x_j'(x_i A) & \text{if } (i,j) \notin E \text{ , } x_i AB'x_j' > r_2 \\ 0 & \text{otherwise} \end{array}$$

The update of the parameters is done consequently as follows:

$$A^{t+1} = A^t - \eta_t \bigtriangledown L_t(A)$$
$$B^{t+1} = B^t - \eta_t \bigtriangledown L_t(B)$$

where η_t is the learning rate at the iteration t.

There are many applications for an approach based on latent spaces. First, they speed up similarity computation, make it independent of graph size, which is useful when a quick result is needed at query time. Moreover, they can be used for link prediction for new nodes which are not yet rich in link structure features. Besides, modeling the graph structure based on the nodes' attributes gives us valuable insights about how attributes relate to the graph formation and evolution.

Transfer learning between networks or from a network to another task is challenging. Learning the latent space on nodes' attributes makes transfer learning between tasks with (approximately) same attributes possible, and this is discussed in more detail in Chapter 4.

3.4.2 Learning From Random Walk Scores

We showed how to learn the transformation from features to the latent space on the network's connections. There is one implicit assumption in the method we explained above, namely that all edges in the network are complete and correct. This assumption is almost always inaccurate, as real-world networks often have spurious and missing links. This assumption is weaker specially when the network is bigger. Moreover, not all links have the same importance and, in weighted networks, links may have different weights. Considering the difference between weights of the links in the process of learning a latent space as explained above is not straightforward.

To solve these problems, we can use the random walk scores and represent them in a latent space. These scores consider all paths between two nodes to score their proximity. In this case we can recognize the missing links, and also the links for which we have less evidence. Moreover, the weights of the links are reflected in the final random walk scores.

For a given network, let the matrix $VP = \begin{pmatrix} VP_{11} & \dots & VP_{1N} \\ & \cdots & \\ VP_{N1} & \dots & VP_{NN} \end{pmatrix}$ gather the symmetric *VP* scores between all nodes in the network. The first attempt to learn the latent space from these scores might be learning all values of VP_{ij} in this matrix by using a transformation to the latent space. This matrix is very huge, $N \times N$, and learning on all values is not applicable. We remember however from Section 3.3.2 that in each row only a few elements are not close to zero, therefore, not all values in this matrix are valuable for learning.

Hence, we use the same learning algorithm that we explained above, but with one difference: instead of training on all the network's edges, we train only on high *VP* score pairs. The high score pairs can be found efficiently by the algorithms also explained above, in Sections 3.3.2 and 3.3.1.

3.4.3 Experimental Results using Latent Space Model

In this section, similarly to Section 3.3.3 above, we study the effectiveness of the latent space model for the link prediction task. As above, we perform ten-fold cross validation on all links in three different networks. To measure the performance of each method, for a given query we sort the rest of the nodes based on their similarity in the latent space, and report precision and recall at 5. The latent space dimension is set to 50.

Table 3.7 shows the average precision and recall for the two latent space models we explained in the previous section, trained on the networks' edges and *VP* scores.

The first important observation is that learning from *VP* scores improves the performance in comparison to learning from networks' edges directly. We discussed the possible reasons in the previous section, including the fact that the training set is improved because the networks'

Learning method	Cora Precision/Recall	CiteSeer Precision/Recall	WebKB Precision/Recall
On Edges model one	2.51/9.61	4.43/18.08	4.62/17.80
On Edges model two	3.65/14.15	5.97/24.44	4.75/20.08
On *VP* 2-NN model one	4.06/16.76	4.97/21.82	5.33/23.69
On *VP* 2-NN model two	5.04/20.6	5.46/24.03	5.66/24.70
On *VP* comm. model one	4.78/19.54	6.16/26.12	6.27/27.67
On *VP* comm. model two	**5.98/24.37**	**6.34/26.96**	6.48/28.13
VP (Without Learning)	5.17/ 20.66	3.10/12.96	**7.26/31.56**

Table 3.7: Precision and recall for the two latent space models trained on networks' edges: *VP*-based community and *VP*-based K-Nearest Neighbors; the highest scores are in bold

edges set is neither complete nor completely correct.

Learning on the community of nodes according to the *VP* scores (Section 3.3.2) has the highest performance. Intuitively we can understand this result: the nodes outside the community are not visible at all, and therefore, it is the best possible configuration to learn a discriminative model.

By learning on *VP* scores we make use of two sources of information, first the networks' structure and then consequently the node's attributes. We showed learning on *VP* scores improves the performance in comparison to the learning directly on networks' edges. Therefore, considering global networks' structure is more effective than local properties.

Moreover, learning on attributes from *VP* scores improves the performance in comparison to using *VP* scores themselves, over the Cora and CiteSeer networks. Therefore, using both sources of information by learning on attributes from *VP* scores is an effective way to model the network.

We observe that in case of WEBKB network, *VP* itself has clearly the highest performance. In other words, the entire networks' structure cannot be learned (explained) by the node's attributes. Nodes' attributes in WEBKB network are derived from the words in each webpage which are not enough alone to predict link formation. In other networks similar situation might happen due to the incomplete set of node's attributes. For example, in most online social networks, people's profiles are not complete and therefore the network's structure can not be fully explained using the node's attributes. In Chapter 6 we design an algorithm to answer this problem.

4 Transfer Learning from Hypertext Encyclopedia to Text Analysis Tasks

In this chapter, we apply the techniques we developed in the previous chapter to transfer knowledge from hypertext encyclopedia towards text analysis tasks. We propose methods for computing semantic relatedness between words or texts by using knowledge from hypertext encyclopedias such as Wikipedia. A network of concepts is built by filtering the encyclopedia's articles, each concept corresponding to an article. Two types of weighted links between concepts are considered: one based on the hyperlinks between the texts of the articles, and another one based on the lexical similarity between them.

To transfer learning from the network of concepts to text analysis tasks we develop two common representation methods: first, a given text is mapped to the corresponding concepts in this network and then to compute similarity between two texts *VP* similarity is applied to compute the distance between sets of nodes. In other words, the shared representation space is the set of concepts in the network and every text is represented in this space.

The second method uses the latent space model explained in Chapter 3, Section 3.4, as the shared representation, and a transformation from words to a latent space is trained over *VP* scores. Therefore, to transfer knowledge from the network to any machine learning algorithm, the given text is transformed using the learned transformation to the latent space.

To evaluate the proposed distance, we apply our method to four important tasks in natural language processing: word similarity, document similarity, document clustering, and document classification, along with a ranking task for information retrieval. The performance of our method is state-of-the-art or close to it for all the tasks, thus demonstrating the generality of the method and the accompanying knowledge resource. Moreover, we show that using both hyperlinks and lexical similarity links improves the scores with respect to a method using only one of them, because hyperlinks bring additional real-world knowledge not captured by lexical similarity. Additionally, the proposed method is applied to Idiap's just-in-time information retrieval system (ACLD), and brings improvement in terms of the relevance of results.

4.1 Introduction to Text Similarity

Estimating the semantic relatedness of two text fragments – such as words, sentences, or entire documents – is important for many natural language processing or information retrieval applications. For instance, semantic relatedness has been used for spelling correction [Budanitsky and Hirst, 2006], word sense disambiguation [Patwardhan et al., 2003, Kohomban and Lee, 2005], or coreference resolution [Ponzetto and Strube, 2007]. It has also been shown to help inducing information extraction patterns [Stevenson and Greenwood, 2005], performing semantic indexing for information retrieval [Baziz et al., 2005], or assessing topic coherence [Newman et al., 2010].

Existing measures of semantic relatedness based on lexical overlap, though widely used, are of little help when text similarity is not based on identical words. Moreover, they assume that words are independent, which is generally not the case. Other measures, such as PLSA or LDA, attempt to model in a probabilistic way the relations between words and topics as they occur in texts, but do not make use of structured knowledge, now available on a large scale, to go beyond word distribution properties. Therefore, computing text semantic relatedness based on concepts and their relations, which have linguistic as well as extra-linguistic dimensions, remains a challenge especially in the general domain and/or over noisy texts.

We propose to compute semantic relatedness between sets of words using the knowledge enclosed in a large hypertext encyclopedia, with specific reference to the English version of Wikipedia used in the experimental part. We propose a method to exploit this knowledge for estimating conceptual relatedness following a statistical approach, by making use of the large-scale, weakly structured knowledge embodied in the links between concepts. The method starts by building a network of concepts under the assumption that every encyclopedia article corresponds to a concept node in the network. Two types of links between nodes are constructed: one by using the original hyperlinks between articles, and the other one by using lexical similarity between the articles' content (Section 4.3).

4.2 Semantic Relatedness: Definitions and Issues

Two samples of language are said to be semantically related if they are about things that are associated in the world, i.e. bearing some influence one upon the other, or being evoked together, in speech or thought, more often than other things. Semantic relatedness is a multi-faceted notion, as it depends on the scale of the language samples (words vs. texts) and on what exactly counts as a relation. In any case, the adjective 'semantic' indicates that we are concerned with relation between the senses or denotations, and not, e.g., surface forms or etymology.

4.2.1 Nature of Semantic Relations for Words and Texts

Semantic relations between words, or rather between their senses, have been well studied and categorized in linguistics. They include classical relations such as synonymy (identity of senses, e.g. ‘freedom’ and ‘liberty’), antonymy (opposition of senses such as ‘increase’ vs. ‘decrease’), hypernymy or hyponymy (e.g. ‘vehicle’ and ‘car’), and meronymy or holonymy (part-whole relation such as ‘wheel’ and ‘car’). From this point of view, semantic similarity is more specific than semantic relatedness. For instance, antonyms are related, but not similar. Or, following Resnik [1995], ‘car’ and ‘bicycle’ are more similar (as hyponyms of ‘vehicle’) than ‘car’ and ‘gasoline’, though the latter pair may seem more related in the world. Classical semantic relations are listed in hand-crafted lexical or ontological resources, such as WordNet [Fellbaum, 1998] or Cyc [Lenat, 1995], or implicitly in Roget’s Thesaurus (as used by Jarmasz [2003]), or they can be inferred from distributional data as discussed below.

Additional types of lexical relations have been described as ‘non-classical’ by Morris and Hirst [2004], for instance based on membership in similar classes (e.g. positive qualities), or on association by location, or due to stereotypes – but these relations do not qualify as similarity ones, and are generally not listed in lexical resources. Budanitsky and Hirst [2006] point out that semantic ‘distance’ can be seen as the contrary of either similarity or relatedness. In this chapter, our use of ‘distance’ will refer to our measure of semantic relatedness as defined below.

At the sentence level, semantic relatedness can subsume notions such as paraphrase or logical relations (e.g., entailment or contradiction). More generally, two sentences can be related by a similarity of topic, a notion that applies to multi-sentence texts as well, even though the notion of ‘topic’ is difficult to define. Topicality is often expressed in terms of the continuity of themes, i.e. referents or entities about which something is predicated, which ensures the coherence of texts. Linguists have analyzed coherence as being maintained by cohesive devices [Hobbs, 1983], which include identity-of-reference, lexical cohesion, and similarity chains based on classical lexical relations [Halliday and Hasan, 1989].

A key relation between the semantic relatedness of words and their occurrence in texts has long been exploited by researchers in natural language processing (NLP) under the form of distributional measures [Mohammad and Hirst, 2005], despite certain limitations pointed out by Budanitsky and Hirst [2006, Section 6.2]. The assumption that sentences and texts form coherent units makes it indeed possible to infer word meanings and lexical relations from distributional similarity [Weeds, 2003], using vector-based models such as Latent Semantic Analysis [Deerwester et al., 1990], possibly enhanced with syntactic information [Padó and Lapata, 2007]. In return, the hidden topical parameters that govern the occurrences of words can be modeled probabilistically (e.g. using PLSA [Hofmann, 1999] or LDA [Blei et al., 2003]), thus providing measures of text similarity.

4.2.2 Use of Encyclopedic Knowledge for Semantic Relatedness

Semantic relatedness has been mainly considered from the perspective of repertoires of semantic relations (often hand-crafted), or from the perspective of relations inferred from the distributional properties of words in collections of texts. However, the computation and use of relations founded on real-world knowledge has been considerably less explored, as it was made possible only recently by the emergence of large-scale hypertext encyclopedias such as Wikipedia. We believe that the use of encyclopedic knowledge may significantly complement semantic relatedness measures based on word distributions only: in the remainder of this section we briefly frame encyclopedic knowledge, outline our proposal, and discuss its task-based validation.

Encyclopedias are lists of general concepts and named entities, accompanied by descriptions in natural language. They differ from dictionaries as they describe concepts or entities, rather than define words, and provide significant factual knowledge for grounding them in the real world, rather than linguistic information only. While printed encyclopedias already include certain references from one entry to another, the linking mechanism is used much more extensively within hypertext encyclopedias such as Wikipedia. As a result, hypertext encyclopedias seem quite adept at capturing semantic relations between concepts, which range from culture-specific to universal ones, including classical and non-classical relations mentioned above.

4.3 Wikipedia as a Network of Concepts

4.3.1 Concepts = Nodes = Vertices

We built our concept network from Wikipedia by using the Freebase Wikipedia Extraction (WEX) dataset Metaweb Technologies [2010] (version dated 2009-06-16). Not all Wikipedia articles were considered appropriate to include in the network of concepts, for reasons related to their nature and reliability, but also to the tractability of the overall method, given the very large number of pages in the English Wikipedia. Therefore, we removed all Wikipedia articles that belonged to the following name spaces: Talk, File, Image, Template, Category, Portal, and List, because these articles do not describe concepts, but contain auxiliary media and information that do not belong into the concept network. Also, disambiguation pages were removed as well, as they only point to different meanings of the title or of its variants.

As noted by Yeh et al. [2009], short articles are often not appropriate candidates to include in the concept network, for several reasons: they often describe very specific concepts which have little chances to occur in texts; they might correspond to incomplete articles (stubs); they contain an unreliable selection of hyperlinks; and their number considerably slows down computation in the network. In previous work, Yeh et al. [2009] set a size limit of 2,000 non-stop words below which entries were pruned, and this limit decreased considerably the size of their network. As our goal is to minimize the risk of removing potentially useful concepts, and to respect as much as possible the original contents of Wikipedia, we set a cut-off limit of 100 non-stop words, thus pruning only very minor articles. This value is thus much lower than the value used by Yeh et al. [2009], and is similar to the one used by Gabrilovich and Markovitch [2009]. Out of an initial set of 4,327,482 articles in WEX, filtering removed about 70% of all articles based on namespaces and length cut-off, yielding a resulting set of 1,264,611 concepts.

Each concept has a main Wikipedia name, which is the title of the main page describing the concept. However, in many cases, other words or phrases can be used as well to refer to the concept. One such type of words can be determined by examining Wikipedia redirects, i.e. articles that have no content but point the user to a proper article with an alternative title. The titles of redirect pages were added as secondary titles to the titles of the articles they redirect to. In addition, for every hyperlink from one article to another, we extracted the corresponding anchor text and considered it as another possible secondary title for the linked article, thus capturing a significant part of the terminological variation of concept names (with some noise due to variability in linking practice). Therefore, each concept has three types of titles (see summary of data structure in Table 4.1): the original one, the anchor texts of the hyperlinks targeting it, and the variants provided by redirect pages, each specific title being listed only once.

4.3.2 Relations = Links = Edges

Relations between concepts can be determined in several ways. In a previous study [Yazdani and Popescu-Belis, 2010], we considered four types of links between concepts: hyperlinks and links computed from similarity of content, of category, and of template. While each type of links captures some form of relatedness, we focus in the present chapter (based on [Yazdani and Popescu-Belis, 2013]) on the first two types, which are the most complementary. However, the proposed computational framework is general enough to accommodate more than two types of links, in particular if an optimal combination can be learned from training data.

The use of *hyperlinks between Wikipedia articles* embodies the somewhat evident observation that every hyperlink from the content of an article towards another one indicates a certain relation between the two articles. These are encyclopedic or pragmatic relations, i.e. between concepts in the world, and subsume semantic relatedness. In other words, if article A contains a hyperlink towards article B, then B helps to understand A, and B is considered to be related to A. Such links represent a substantial amount of human knowledge that is embodied in the Wikipedia structure. It must be noted that these links are essentially asymmetric, and we decided to keep them as such, i.e. to list for a given page only its outgoing links and not the incoming ones. Indeed, observations showed that while the target page of a link helps understanding the source one, the contrary is not always true or the relation is not specific enough. For each article, the XML text from WEX was parsed to extract hyperlinks, resulting in a total of 35,214,537 hyperlinks – a time-consuming operation that required also the ability to handle instances of ill-formed XML input.

The second type of links is based on *similarity of lexical content between articles* of Wikipedia, computed from word co-occurrence. If two articles have many words in common, then a topic-similarity relation holds between them. To capture content similarity, we computed the lexical similarity between articles as the cosine similarity between the vectors derived from the articles' texts, after stopword removal and stemming using Snowball.[1] We then linked every article to its k most similar articles, with a weight according to the normalized lexical similarity score (for non-zero weights). In the experiments described below, k was set to 10, so that each node has ten outgoing links to other nodes, based on lexical similarity.[2] The value of $k = 10$ was chosen to ensure computational tractability, and is slightly lower than the average number of hyperlinks per concept, which is about 28. As the Wikipedia articles are scattered in the space of words, tuning k does not seem to bring crucial changes. If k is very small then the neighborhood contains little information, whereas a large k makes computation time-consuming.

Concept ID	Integer
Names of the concept	Name of article in the encyclopedia
	Alternative names redirecting to the article
	Anchor texts of incoming hyperlinks
Description of the concept	Text
Relations to other concepts (outgoing links)	Hyperlinks from the description towards other concepts (no weights)
	Lexical similarity links to the ten closest concepts (weights from cosine similarity)

Table 4.1: Logical structure of each node in the network resulting from the English Wikipedia.

4.3.3 Properties of the Resulting Network

The processing of the English Wikipedia resulted in a very large network of concepts, each of them having the logical structure represented in Table 4.1. The network has more than 1.2 million nodes (i.e. vertices), with an average of 28 outgoing hyperlinks per node and 10 outgoing content links per node.

A natural question arising at this point is: how can the structure of the network be characterized, apart from putting it to work? It is not possible to visualize the entire network due to its size, and displaying only a small part, as done for instance by Coursey et al. [2009, Figure 1], might not be representative of the entire network.[3]. A number of quantitative parameters have been proposed in graph theory and social network analysis, and some have for instance been used to analyze WordNet (and an enhanced version of it) by Navigli and Lapata [2010].

[1] From the Apache Lucene indexing system available at http://lucene.apache.org/.

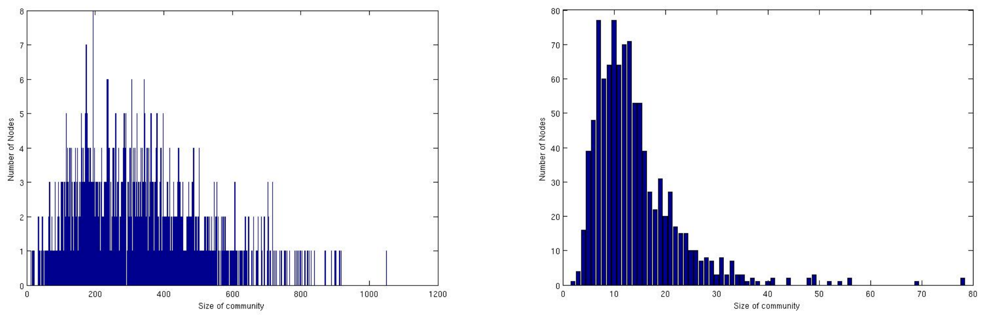
[2] Therefore, the outdegree of all nodes is 10, but as the indegree may vary (the number of incoming links), the graph is not strictly speaking a 10-regular graph.

[3] An example of neighborhood according to our relatedness measure is however shown in Section 4.4.1

We compute below some well-known parameters for our network, and add a new, more informative characterization.

A first characteristic of graphs is their degree distribution, i.e. the distribution of the number of direct neighbors per node. For the original Wikipedia with hyperlinks, a representation [Grishchenko, 2008] suggests that the distribution follows a power law. A more relevant property here is the network clustering coefficient, which is the average of clustering coefficients per node, defined as the size of the immediate neighborhood of the node divided by the maximum number of links that could connect all pairs of neighbors [Watts and Strogatz, 1998]. For our hyperlink graph, the value of this coefficient is 0.16, while for the content link graph it is 0.26.[4] This shows that the hyperlink graph is less clustered than the content link one, i.e. the distribution of nodes and links is more homogeneous, and that overall the two graphs have rather low clustering.[5]

We propose an ad-hoc measure offering a better illustration of the network's topology, aimed at finding out whether the graph is clustered or not – i.e., whether the communities of nodes based on neighborhoods have a preferred size, or are uniformly distributed. We consider a sample of 1000 nodes. For each node of the graph, the Personalized PageRank algorithm [Haveliwala, 2003] is initialized from that node and run, thus resulting into a proximity coefficient for each node in the graph to the initial node. This is first done using hyperlinks, and then using the content links. The community size for the node is computed by sorting all nodes with respect to their proximity and counting how many nodes contribute to 99% of the mass.



(a) Hyperlink graph

(b) Content link graph

Figure 4.1: Distribution of community sizes for a sample of 1000 nodes (see text for the definition of a community). For each community size (x-axis) the graphs show the number of nodes (y-axis) having a community of that size, for each of the two graphs built from Wikipedia. Both graphs have a tendency towards clustering, i.e. a non-uniform distribution of links, with an average cluster size of 150–400 for hyperlinks and 7–14 for content links.

[4]For the content links, the coefficient was computed regardless of their weights. A recent proposal for computing it for weighted graphs could be applied too [Opsahl and Panzarasa, 2009].

[5]The observed values, together with the power law degree distribution, suggest that our graph is a scale-free network – characterized by the presence of "hub" nodes – or a small-world network [Watts and Strogatz, 1998]. However, it is not clear what impact these properties defined mainly for social networks would have here.

The results shown in Figure 4.1 show that the distribution is neither flat nor uniformly decreasing, but has a peak, which provides an indication of the average size of clusters. This size is around 150–400 nodes for the hyperlink graph, without a sharp maximum, showing less clustering than for content links, for which this average is around 7–14 nodes. The latter value is partly related to the 10-link limit set for content links, but is not entirely due to it, as the limit concerns only outgoing links. The use of hyperlinks thus avoids local clusters and extends considerably the connectivity of the network in comparison to content similarity ones.

4.4 Common Representation Space for Transfer Learning

In order to transfer human knowledge embodied in the Wikipedia network of concepts towards a measure of text similarity in various text analysis tasks, we need to build a shared representation for text fragments. We propose two shared representation models (i.e. spaces), which we explain in Sections 4.4.1 and 4.4.2 below.

The network of concepts built from Wikipedia consists of many concepts from human knowledge. Therefore, the set of concepts in the network is rich enough so that it can represent the content of any text fragment. The first shared representation we develop is the set of concepts in the network: a given text is simply mapped to the corresponding concepts in this network. Then, to compute similarity between two texts, *VP* similarity is applied to compute the distance between the two sets of nodes (concepts).

The second method uses the latent space model that we explained in Section 3.4 as the shared representation. In this approach, we assume that there is a latent space in which semantically similar texts are placed in close positions and semantically unrelated texts are placed farther away from each other. We showed that each concept in the network (corresponding to a Wikipedia article) has a text body which explains the concept. We learn a transformation from words in the title and body to the latent space so that two similar concepts in terms of *VP* are in close distance. Therefore, to transfer knowledge from the network to any processing method that uses feature vectors, texts are transformed using the learned transformation into the latent space. We now explain each approach in more detail.

4.4.1 Mapping Text Fragments to Concepts in the Network

For mapping, two cases must be considered, according to whether the text matches exactly the title of a Wikipedia page or not. Exact matching is likely to occur with individual words or short phrases, but not with entire sentences or longer texts.

If a text fragment consists of a single word or a phrase that *matches exactly the title of a Wikipedia page*, then it is simply mapped to that concept. In the case of words or phrases that may refer to several concepts in Wikipedia, we simply assign to them the same page as the one assigned by the Wikipedia contributors as the most salient or preferred sense or denotation.

For instance, ‘mouse’ directs to the page about the animal, which contains an indication that the ‘mouse_(computing)’ page describes the pointing device, and that other senses are listed on the ‘mouse_(disambiguation)’ page. So, here, we simply map ‘mouse’ to the animal concept. However, for other words, no sense or denotation is preferred by the Wikipedia contributors, e.g. for the word ‘plate’. In such cases, a disambiguation page is associated to that word or phrase. We chose not to include such pages in our network, as they do not correspond to individual concepts. So, in order to select the referent page for such words, we simply use the lexical similarity approach we will now describe.

When a fragment (a word, phrase, sentence, or text) *does not match exactly the Wikipedia title of a vertex in our network*, it is mapped to the network by computing its lexical similarity with the text content of the vertices in the network, using cosine distance over stemmed words, stopwords being removed. Concept names (principal and secondary ones) are given twice as much weight as the words found in the description of the concept. The text fragment is mapped to the k most similar articles according to this similarity score, resulting in a set of at most k weighted concepts. The weights are normalized, summing up to one, therefore the text representation in the network is a *probability distribution over at most k concepts*. Finding the k closest articles according to lexical similarity can be done efficiently using the Apache Lucene search engine (see note 1).

For example, consider the following text fragment to be mapped to our network: “Facebook: you have some serious privacy and security problems.” When this fragment is mapped to the $k = 10$ most similar Wikipedia articles, the resulting probability distribution is the following one: ‘Facebook’ (0.180), ‘Facebook Beacon’ (0.119), ‘Facebook history’ (0.116), ‘Criticism of Facebook’ (0.116), ‘Facebook features’ (0.116), ‘Privacy’ (0.084), ‘Privacy International’ (0.080), ‘Internet privacy’ (0.080), ‘Privacy policy’ (0.054), ‘Privacy Lost’ (0.054).

This mapping algorithm has an important role in the performance of the final system, in combination with the network distance (*VP*). It must however be noted that the effects of wrong mappings at this stage are countered later on. For instance, when large sets of concepts related to two text fragments are compared, a few individual mistakes are not likely to alter the overall relatedness scores. Alternatively, when comparing individual words, wrong mappings are less likely to occur because the test sets for word similarity described in Section 4.6, page 60, also consider implicitly the most salient sense of each word, just as described above for Wikipedia.

VP Between Two Sets of Nodes

After mapping to one or more Wikipedia articles, as just explained, each text is thus represented by a weighted set of nodes. In order to measure the similarity between two texts after mapping, we will generalize the definition of *VP* so that it measures the similarity between two weighted set of nodes.

We first generalize the definition of *VP* between two nodes given in Section 3.1 to a measure from a distribution to a node. Given a probability distribution $\vec{r}$ over concepts (i.e. a weighted set of concepts), and a concept s_j in the network, we first compute the probability of visiting s_j for the first time when a random walker starts from $\vec{r}$ in the network, i.e. from any of the concepts in $\vec{r}$ that have a non-zero probability. To compute visiting probability, the following procedure provides a model of the state S_t of the random walker, i.e. the concept in which the random walker is positioned. Executing the procedure until termination gives the visiting probability *VP*.

Step 0: Choose the initial state of the walker with probability $P(S_0 = s_i|\vec{r}) = r_i$. In other words, position the random walker on any of the concepts of $\vec{r}$ with the probability stated in $\vec{r}$.

Step t: Assuming S_{t-1} is determined, if $S_{t-1} = s_j$ then return 'success' and finish the procedure. Otherwise, with probability α, choose a value for the next concept S_t according to the transition matrix C, and with probability $1-\alpha$, return 'fail'. The possibility of 'failing', or absorption probability, introduces a penalty over long paths that makes them less probable.

Similarly we introduce C' as being equal to the transition matrix C, except that in row j, $C'(j,k) = 0$ for all k. We remind that the transition matrix C, is the linear combination of the transition matrices of hyperlinks and content links, which they are built from the weights of links in the network. The possible weights are 0 or 1 for the hyperlink matrix, and actual lexical similarity scores (or 0) for the content similarity matrix.

The probability of success in the process, i.e. the probability of visiting s_j starting from $\vec{r}$, is the sum over all probabilities of success with different lengths:

$$p(success) = \sum_{t=0}^{\infty} p^t(success) = \sum_{t=0}^{\infty} \alpha^t (\vec{r}C'^t)_j.$$

Therefore, the computation is the same as before, the only difference here is that $\vec{r}$ is resulting from the mapping algorithm and shows a set of initial concepts instead of one node.

To compute the *VP* from a weighted set of concepts to another set, i.e. from distribution $\vec{r}_1$ to distribution $\vec{r}_2$, we construct a virtual node representing $\vec{r}_2$ in the network, noted $s_R(\vec{r}_2)$. We then connect all concepts s_i to $s_R(\vec{r}_2)$ according to the weights in $\vec{r}_2$. We now create the transition matrix C' by adding a new row (numbered n_R) to the transition matrix C with all elements zero to indicate $s_R(\vec{r}_2)$, then adding a new column with the weights in $s_R(\vec{r}_2)$, and updating all the other rows of C as follows (C_{ij} is an element of C):

$C'_{ij} = C_{ij}(1 - (\vec{r}_2)_i)$ for $i, j \neq n_R$
$C'_{in_R} = (\vec{r}_2)_i$ for all i
$C'_{n_R j} = 0$ for all j.

These modifications to the graph are local and can be done at run time, with the possibility to undo them for subsequent text fragments to be compared. The algorithm to compute *VP* is identical to the section 3.1 and therefore, the same algorithms and properties we explained there hold for this application here as well.

To compute relatedness between two texts, we average between the visiting probability of $\vec{r}_1$ given $\vec{r}_2$ (noted $VP(\vec{r}_2, \vec{r}_1)$) and the visiting probability of $\vec{r}_2$ given $\vec{r}_1$ (noted $VP(\vec{r}_1, \vec{r}_2)$). A larger value of the measure indicates closer relatedness.

Illustration of Visiting Probability *'to'* vs. *'from'* a Text

We showed earlier in Section 3.1.3 the importance of the symmetric *VP*. Here we illustrate in a similar way the difference between *VP from* and *to* and justify the choice of symmetric *VP*. To illustrate the difference between *VP to* and *from* a text fragment, we consider the following example. Though any text fragment could be used, we took the definition of 'jaguar' (animal) from the corresponding Wikipedia article. Although the word 'jaguar' is polysemous, the topic of this particular text fragment is not ambiguous to a human reader. The text was first mapped to Wikipedia as described above. Then, using *VP* with equal weights on hyperlinks and content links, the ten closest concepts (i.e. Wikipedia pages) in terms of *VP* from the text, and respectively to it, were found to be the following ones:

- Original text: "The jaguar is a big cat, a feline in the Panthera genus, and is the only Panthera species found in the Americas. The jaguar is the third-largest feline after the tiger and the lion, and the largest in the Western Hemisphere."
- Ten closest concepts according to their *VP to* the text: 'John Varty', 'European Jaguar', 'Congolese Spotted Lion', 'Lionhead rabbit', 'Panthera leo fossilis', 'Tigon', 'Panthera hybrid', 'Parc des Félins', 'Marozi', 'Craig Busch'.
- Ten closest concepts according to their *VP from* the text: 'Felidae', 'Kodkod', 'North Africa', 'Jaguar', 'Panthera', 'Algeria', 'Tiger', 'Lion', 'Panthera hybrid', 'Djémila'.

The closest concepts according to *VP from* a text tend to be more general than closest concepts according to *VP to* the text. For instance, the second list above includes the genus and family of the jaguar species (Panthera and Felidae), while the first one includes six hybrid or extinct species related to the jaguar. Concepts close *to* a text thus bring detailed information related to the topics of the text, while concepts close *from* a text are more popular and more general

Wikipedia articles. Note also that none of the closest concepts above is related to the Jaguar car brand, as found by examining each page, including the lesser-known ones (both persons cited are wildlife film makers and park founders). However, not all concepts are related in an obvious way (e.g. 'Algeria' is likely retrieved through 'Western Hemisphere').

The behavior illustrated on this example is expected given the definition of *VP*. A concept that is close *to* a text has more paths in the network towards the text with respect to other concepts in the network, which means that the concept is quite specific in relation to the topics in the text, as in the above example. Conversely, a concept that is close *from* a text (in terms of *VP* from a text) is typically related by many paths from the text to the concept, in comparison to other concepts. This generally means that it is likely that the article is a general and popular article around the topics of the text. If we consider the hierarchy between concepts from more general to more specific ones, then concepts close *to* a text are generally lower in the hierarchy with respect to the text, and concepts close *from* a text are generally higher.

4.4.2 Learning Embeddings to Latent Space

The second method we propose to measure text similarity using *VP* is to learn an embedding (transformation) from words to a latent space using as a criterion the *VP* scores on the Wikipedia concept network, following the general approach introduced in Section 3.4 above.

At training time, given a series of samples – that is, pairs of texts with *VP* values from the first text to the second one – the goal is to learn a transformation from the space of words to a latent space, so that the similarity between the latent representation of the texts is as close as possible to the *VP* similarity. In other words, the goal is to approximate *VP* between two texts i and j by the matrix product $x_i AB' x'_j$, where x_i and x_j are the TF-IDF vectors of the two texts constructed from their words using a fixed dictionary. The size of matrices A and B is $n \times m$, with n being the size of the dictionary (number of words) and m the size of the latent space (akin to the number of topics in topic models). Two different matrices A and B are needed because *VP* values are not symmetric in i and j.

In principle, all pairs of Wikipedia articles (i.e., texts) corresponding to nodes in our network can be used for training, but this set is extremely large (ca. 1.4×10^{12}) and moreover, we showed that the most values are close to zero and are not valuable for training. Therefore, similar to the section 3.4 we formulate the following constraints for training: (1) training should focus on neighboring articles (articles with high *VP* values), and (2) the exact values of *VP* are replaced with the ranking of pairs of articles by decreasing *VP*. We show here that under these constraints valuable embeddings can be learned.

Let $VPto_k(i)$ be the set of the k closest articles *to* the article i according to *VP* similarity. We define a hinge loss function L as follows, so that the similarity between i and its k closest articles is larger than the similarity to all other articles by a fixed margin M.

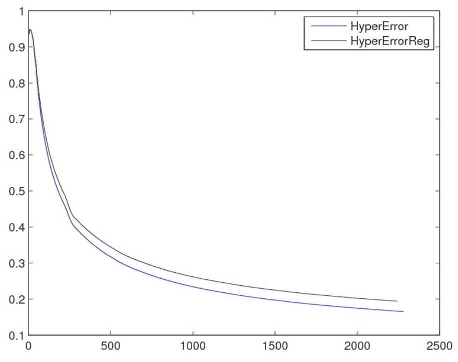
$$L = \sum_{i \in \mathrm{WP}} \sum_{j \in VPto_k(i)} \sum_{z \notin VPto_k(i)} max(0, M - x_i AB' x_j' + x_i AB' x_z')$$

We optimize L with stochastic gradient descent: in each iteration we randomly choose one article i, then randomly choose one of the k closest articles to i (noted j) and one other article from the rest of documents (noted z).

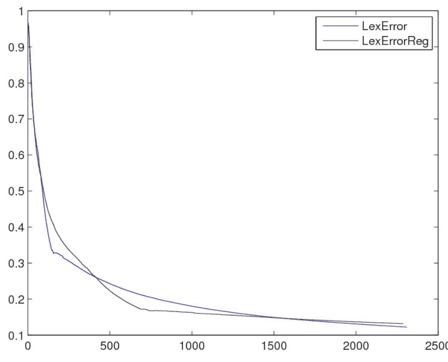
In our experiments, we set $k = 10$ and $M = 0.2$. We computed the $VPto_k(i)$ values for all i using the algorithms in chapter 3. We built the dictionary by using the Snowball tokenizer from Lucene and removing the highest and lowest frequency words, keeping around 60,000 words. We set the number of topics to $m = 50$, because a larger m offers a higher learning capability, but also increases linearly the number of parameters to learn. Given the size of the training set, we chose a rather small number m to make the training possible in a reasonable time, and found a satisfactory prediction error.

Moreover, to perform regularization over matrices A and B when optimizing L, we impose the constraint that A and B are orthonormal. In order to apply this constraint, we project at every 1000 iterations both A and B to their nearest orthogonal matrix found by using SVD decomposition. The rationale for the constraint is the following: if we assume that each latent dimension corresponds to a possible topic or theme, then these should be as orthogonal as possible.

Figures 4.2(a) and 4.2(b) show the average training error at every 1000 iterations, with and without regularization, respectively for the hyperlink graph and for the content link one. The training error is computed as the number of text triples (i, j, z) for which the test in the hinge loss function is false, i.e. $x_i AB' x_j' - x_i AB' x_z' < M$.



(a) Hyperlink graph



(b) Content link graph

Figure 4.2: Average training error for learning embeddings, measured every 1000 iterations, with and without using regularization.

To test the predictive power of the embeddings as a replacement for the computation of *VP* at runtime, we excluded 50,000 articles from the training set, and then built 50,000 triples of articles by choosing randomly, for each of the excluded articles, one article from the k closest ones and one randomly from the rest of the articles. The test set error, which is the number of triples from this set that do not respect the order given by *VP* similarities, is shown in Table 4.2.

Training set for embedding	**Error (%)**
Hyperlinks	9.8
Hyperlinks with Regularization	13.5
Content Links	5.4
Content Links with Regularization	5.9

Table 4.2: Accuracy of embeddings learned over four different training sets. The error (percentage) is computed for 50,000 triples of articles from a separate test set, as the number of triples that do not respect the ordering of *VP* similarities.

The two main findings are the following. First, *VP* over the hyperlinks graph is harder to learn, which may be due to the fact that hyperlinks are defined by users in a manner that is not totally predictable. Second, regularization decreases the prediction ability. However, if regularization traded prediction power for more generality, in other words if it reduced overfitting to this problem and made the distance more general, then it would still constitute a useful operation. This will be checked in the experiments in Sections 4.8, 4.9 and 4.11. Moreover, these experiments will show that the embeddings can be plugged into state-of-the-art learning algorithms as prior knowledge, improving their performance.

Learning embeddings in comparison to mapping to the concept network has some advantages. The main one is that it can be applied with a much lower cost at run time. The embedding approach does not require a fixed number of nodes to project a document on. Moreover, it can be more easily integrated as prior knowledge to other learning algorithms for NLP, and it can be applied to very large scale problems.

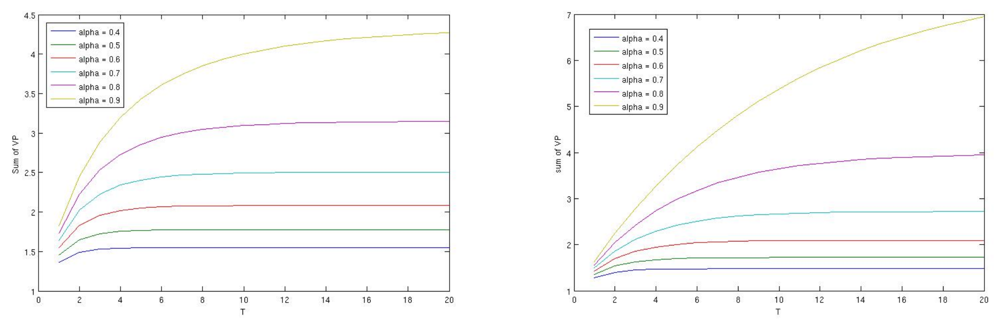
4.5 Empirical Analyses of *VP* and Approximations

We now analyze the convergence of the two approximation methods proposed in section 3.2. In the case of path insensitive ε-truncated approximation lets T shows the maximum path length that is expanded, when T increases to infinity, the path insensitive ε-truncated approximated *VP* converges towards the exact value, but computation time increases linearly with T. In the case of path sensitive ε-truncation, when ε tends to zero the approximated value converges towards the real one, but again computation time increases. Therefore, we need to find the proper values for T and ε as a compromise between the estimated error and the computing time.

4.5.1 Convergence of the path insensitive ε-Truncated *VP* over Wikipedia

The value of T required for a certain level of convergence (upper bound on the approximation error) depends on the transition matrix of the graph. It was not possible to find a convenient theoretical upper bound, so we analyzed the relation between T and the approximation error empirically, by a sampling method. We chose randomly a set S of 1000 nodes in the graph, and computed the truncated *VP* from all nodes in the graph to each of the nodes in S using different T. Then we computed the average of the values of truncated *VP* from all nodes to the nodes in S: $VP_{sum}(T) = \sum_{j\in S}\sum_{i\neq j} VP^T(i,j)/|S|$.

Given that S is a large random sample, we consider that the evolution of $VP_{sum}(T)$ with T is representative of the evolution of an "average" VP^T with T. Figure 4.3(a) shows the values of $VP_{sum}(T)$ depending on T for the Wikipedia graph with the lexical similarity (content) links, for various values of α. Figure 4.3(b) shows the same curves for the Wikipedia graph with hyperlinks. Both analyses show, as expected, that larger values of α correspond to a slower convergence in terms of T, because a larger α requires the random walker to explore longer paths for the same level of approximation. (The exact value toward which $VP_{sum}(T)$ converges is not important here.) The figures give some indication, for each given α, of the extent to which the paths should be explored for an acceptable approximation of the non-truncated *VP* value. In our experiments, we chose $\alpha = 0.8$ and $T = 10$ as a compromise between computation time and accuracy – the corresponding approximation error is less than 10% in Figures 4.3(a) and 4.3(b).



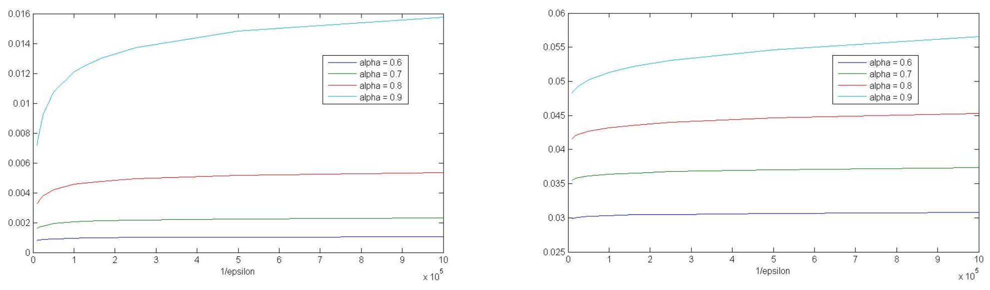
(a) Content link graph

(b) Hyperlink graph

Figure 4.3: $VP_{sum}(T)$ as an "average *VP*" for content links (a) and hyperlinks (b) over the Wikipedia graph, depending on T, for α varying from 0.4 to 0.9 (by 0.1, bottom to top). The shape of the curves indicates the values of T leading to an acceptable approximation of the exact, non-truncated value: $\alpha = 0.8$ and $T = 10$ were chosen in the subsequent experiments.

4.5.2 Convergence of path sensitive ε-Truncated *VP* over Wikipedia

To analyze the error induced by the path sensitive ε-truncation when computing *VP* over the Wikipedia graph, we proceeded in a similar way. We chose 5000 random pairs of nodes and computed the sum of path sensitive ε-truncated *VP* between each pair. Figure 4.4(a) shows the values of the sum depending on $1/\varepsilon$ for the Wikipedia graph with content links, and Figure 4.4(b) for the Wikipedia graph with the hyperlinks. Again, it appears from the curves that for a larger α, smaller values of ε are needed to reach the same level of approximation error, because for a larger α longer paths must be explored to reach the same approximation level. The ε-truncation is used for word similarity and document similarity below, with a value of $\varepsilon = 10^{-5}$, and a value of $\alpha = 0.8$ as above.



(a) Content link graph (b) Hyperlink graph

Figure 4.4: Sum of ε-Truncated *VP* depending on $1/\varepsilon$ for the Wikipedia graph with content links (a) or hyperlinks (b), for α from 0.6 to 0.9 (bottom to top). The shape of the curves indicates the values of ε leading to an acceptable approximation of the non-truncated value: $\varepsilon = 10^{-5}$ and $\alpha = 0.8$ were chosen for subsequent experiments.

4.5.3 Differences between the Random Walk Model over Content Links and Direct Lexical Similarity between Articles

Performing a random walk over the Wikipedia graph leads to a relatedness measure that is different from direct lexical similarity (i.e. cosine similarity in the space of words), as we empirically show here through the following experiment. We randomly chose 1000 nodes and sorted all other nodes (Wikipedia articles) based on the cosine similarity with the randomly selected nodes, measured using TF-IDF vectors. In parallel, for each of the randomly selected nodes, all other nodes were also sorted based on the average *VP* using paths of length at most $T = 10$.

The comparison of the two resulting sorted lists for each node shows the difference between the two measures. To perform this comparison, for each node we look at the intersection of the heads of the two sorted lists (neighborhoods of the node), varying the size of these subsets. Figure 4.5 shows the percentage of nodes in common depending on neighborhood sizes, for different values of α (which changes little to the results). It appears that for the closest neighbors, and in particular for *the* closest one, both lexical similarity and *VP* over content links return the same nodes. However, when expanding the size of the neighborhood, the size of the intersection decreases rapidly. For example, when looking at the ten closest nodes, only about 50% of the nodes on average are the same in the two lists. This is an empirical argument showing that walking over content links leads to a different relatedness measure than simply using lexical similarity between articles.

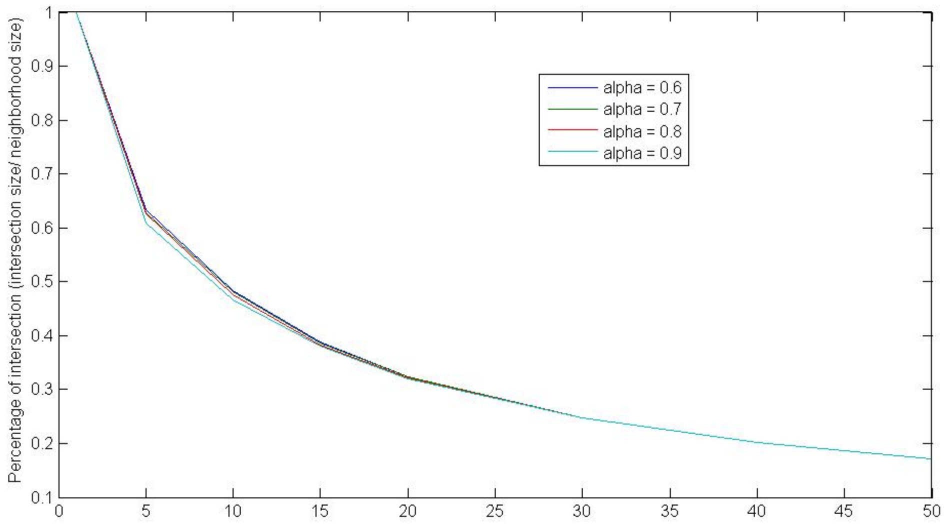


Figure 4.5: Average proportion of identical nodes among K closest neighbors using cosine similarity vs. average T-truncated *VP*, for varying sizes K of the neighborhood and various values of α (which have little influence on the result). The proportion decreases for larger neighborhoods, i.e. the two metrics give quite different results for smaller relatedness values.

4.6 Word Similarity

In the following sections, we assess the effectiveness of visiting probability by applying it to four language processing tasks. In particular, for each task, we examine each type of link separately and then compare the results with those obtained for combinations of links, in the attempt to single out the optimal combinations.

The word similarity task has been heavily researched using a variety of methods and resources – such as WordNet, Roget's Thesaurus, the English Wikipedia or Wiktionary – starting for instance with Resnik's seminal paper [Resnik, 1995] and even earlier. We have reviewed in Chapter 2 some of the recent work on word similarity, focusing mainly on graph-based methods over Wikipedia but also WordNet. Recent studies, including also references to previous scores and several baselines, have been made by Bollegala et al. [2007], Gabrilovich and Markovitch [2007] (see also [Gabrilovich and Markovitch, 2009, Tables 2 and 3]), Zesch et al. [2008], Agirre et al. [2009], and Ramage et al. [2009], among others.

Three test sets for the English word similarity task have been extensively used in the past. They consist of pairs of words accompanied by average similarity scores assigned by human subjects to each pair. Depending on the instructions given to the subjects, the notion of 'similarity' was sometimes rather interpreted as 'relatedness', as discussed for instance by Hughes and Ramage

[2007, Section 5]. The three sets, all of them reproduced in Jarmasz's thesis [Jarmasz, 2003] for instance, were designed respectively by Rubenstein and Goodenough [1965] (henceforth, R&G, 65 pairs, 51 judges), by Miller and Charles [1991] (M&C, 30 pairs), and by Finkelstein et al. [2002] (WordSimilarity-353, with 353 pairs).

We estimate the relatedness between words by mapping them to concepts and computing the path sensitive ε-truncated *VP* distance between them with $\varepsilon = 10^{-5}$. We set the value of $\alpha = 0.8$ as explained in Section 4.5.2 above. The correlation with human judgments of relatedness is measured using the Spearman rank correlation coefficient ρ as well as the Pearson correlation coefficient r between the *VP* values for each pair and the human judgments.

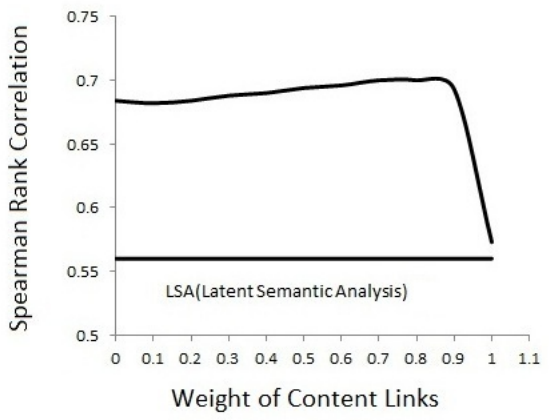


Figure 4.6: Spearman rank correlation ρ between automatic and human judgments of word similarity on the WordSimilarity-353 data set, depending on the weight of content links in the random walk (the weight of hyperlinks is the complement to 1). The best scores, $\rho = 0.70$, are reached when content links have more weight than hyperlinks (0.7–0.8 vs. 0.3–0.2). The result of LSA, $\rho = 0.56$, is quoted from Gabrilovich and Markovitch [2007], and is outperformed by other scores in the literature.

The values of the Spearman rank correlation coefficient ρ on the WordSimilarity-353 data set, for varying relative weights of content links vs. hyperlinks in the random walk, are shown in Figure 4.6. The best scores reach $\rho = 0.70$ for a combination of hyperlinks and content links, weighted between 0.3/0.7 and 0.2/0.8. As shown in the figure, results improve by combining links in comparison with using a single type. Results improve rapidly when hyperlinks are added to content ones (right end of the curve), even with a small weight, which shows that adding encyclopedic knowledge represented by hyperlinks to word co-occurrence information can improve significantly the performance. Conversely, adding content links to hyperlinks also improves the results, likely because the effect of spurious hyperlinks is reduced after adding content links.

To find out whether the two types of links encode similar relations or not, we examined to what extent results using *VP* with hyperlinks only are correlated with results using content links only. The Spearman rank correlation coefficient between these scores is $\rho = 0.71$, which shows that there is some independence between scores.

We also tested our method on the R&G and M&C data sets. The values of the Pearson correlation coefficient r for previous algorithms using lexical resources are given by Jarmasz [2003, Section 4.3.2], by Gabrilovich and Markovitch [2009, Table 3] and by Agirre et al. [2009, Table 7], showing again that for word similarity, using lexical resources is very successful, reaching a correlation of 0.70–0.85 with human judgments. For our own algorithm, correlation r with human judgments on R&G and M&C is, respectively, 0.38–0.42 and 0.46–0.50, depending on the combination of links that is used. Lexically-based techniques are thus more successful on these data sets, for which only the lexical similarity relation is important, while our method, which considers linguistic as well as extra-linguistic relations, is less efficient.

However, if we examine the Spearman rank correlation ρ on the R&G and M&C data sets, considering therefore only the ranking between pairs of words and not the exact values of the relatedness scores, then our method reaches 0.67–0.69. Our scores are thus still lower than the best lexically-based techniques, which have a ρ between 0.74–0.81 and 0.69–0.86 on R&G and M&C, but the difference is now smaller. Our method is thus more suitable for capturing the ranking of the pairs instead of their exact scores, an observation that was also made by Gabrilovich and Markovitch [2009].

Gabrilovich and Markovitch [2007, Table 4] (see also [Gabrilovich and Markovitch, 2009, Table 3]) and Agirre et al. [2009, Table 9] provide the values of the Spearman rank correlation coefficient ρ of previous methods on the WordSimilarity-353 data set. The best results is obtained by Explicit Semantic Analysis with $\rho = 0.75$ and by a system combination of distributional and WordNet-based methods (Agirre et al. [2009]) with $\rho = 0.78$. Apart from these methods, the best reported scores on this data are the ones reached by LSA with $\rho = 0.56$ oly. Our method outperforms this score by a margin that is similar to that of ESA or system combination [Agirre et al., 2009], though our method does not quite reach their scores.

Some authors have attempted to reproduce the ESA scores: Zesch et al. [2008] reached only $\rho = 0.46$, while Ramage et al. [2009] and Hassan and Mihalcea [2009] reported results close to the original ones [Gabrilovich and Markovitch, 2007, 2009]. A key factor that ensures high performance seems to be the cleaning procedure applied to concept vectors [Gabrilovich and Markovitch, 2009, 3.2.3]. Overall, to facilitate comparison of our scores to important scores in the literature, Table 4.3 provides a synthetic view.

To improve our understanding of the types of links, Figure 4.7 shows the average frequency of the path lengths traveled for computing path sensitive ε-truncated *VP* on the word pairs from WordSimilarity-353, for three different combinations of links. The results show that using hyperlinks shortens the length of the average path that is used for the computation of *VP*. Conversely, by using hyperlinks, the number of paths between words is increasing dramatically

WordSimilarity-353		**M&C data set**		
Study	ρ	**Study**	ρ	r
Finkelstein et al. [2002]	0.56	Wu and Palmer [1994]	0.78	0.78
Jarmasz [2003]	0.55	Resnik [1995]	0.81	0.80
Strube and Ponzetto [2006]	0.48	Leacock and Chodorow [1998]	0.79	0.82
Hughes and Ramage [2007]	0.55	Lin [1998]	0.82	0.83
Gabrilovich and Markovitch [2007]	0.75	Jarmasz [2003]	0.87	0.87
Agirre et al. [2009]	0.78	Patwardhan and Pedersen [2006]	N/A	0.91
		Bollegala et al. [2007]	0.82	0.83
		Alvarez and Lim [2007]	N/A	0.91
		Hughes and Ramage [2007]	0.90	N/A
		Agirre et al. [2009]	0.92	0.93
VP	0.70	*VP*	0.69	0.50

Table 4.3: A comparison of several word similarity scores, in terms of Spearman rank correlation ρ and Pearson correlation r, on two data sets: WordSimilarity-353 [Finkelstein et al., 2002] and M&C [Miller and Charles, 1991]. Our scores for *VP* appear in the last line, and are for WordSimilarity-353 in the upper range, though not above state-of-the-art ones.

in comparison to using content links only, a fact that explains why adding hyperlinks, even with a small weight, to content links improves the results so rapidly in Figure 4.6.

4.7 Document Similarity

The estimation of document similarity is another task on which our proposal was assessed. The document similarity data set used in this experiment was gathered by Lee et al. [2005], and contains average human similarity scores for all pairs of a set of 50 documents.

As in the experiment with word similarity (using the same parameters $\alpha = 0.8$ and $\varepsilon = 10^{-5}$), we tested our method using various combinations of weights for the two types of links, with results shown in Figure 4.8. Following Gabrilovich and Markovitch [2007] – and because the averaged human judgments cover only a small range of possible values – we use the Pearson correlation coefficient r to evaluate how close our method approaches the human judgments. For these experiments, each document from the set was mapped to the 1,000 closest concepts in the network. Otherwise, the same random walk parameters as for the word similarity task were used.

Our findings are that the behavior of the system and its best results are similar to the previous experiment on word similarity. Adding hyperlinks to content links improves the correlation sharply, but adding content links to hyperlinks also improves the results (after an initial decrease), so that the best performance ($r = 0.676$) is reached with a combination of links, which appears to be very similar to the word similarity experiment.

The authors of the data set (Lee et al. [2005]) report the results of several other methods on the document similarity task, the best one reaching $r = 0.60$ using LSA. However, in this case (also mentioned by Gabrilovich and Markovitch [2007]), LSA was trained only a small document

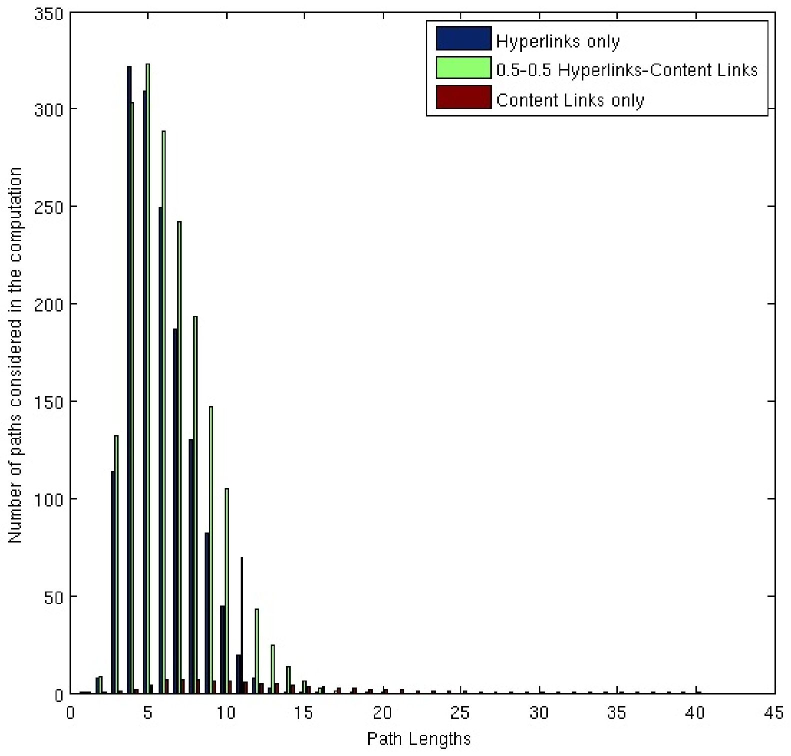


Figure 4.7: Average frequency of the path lengths contributing to *VP* on the WordSimilarity-353 data set, in three configurations (left to right for each integer value): hyperlinks only, equal weights, content links only.

corpus of 314 news articles. When trained over a much larger corpus, the performance of LSA increases to $r = 0.69$, a value reported by Hassan and Mihalcea [2011] after training LSA over the entire Wikipedia corpus. ESA Gabrilovich and Markovitch [2007] reaches a score of $r = 0.72$ on this task, which outperforms all other methods (as in the case of word similarity) including ours, although by a small margin. Indeed, with our method, the best observed combination reached $r = 0.676$. Therefore, although our method is below some of the best results in the literature, it reaches nevertheless high scores with a general semantic relatedness measure.

As we did for word similarity, we show in Figure 4.9 the frequency of the path lengths traveled for computing *VP*, averaged on the document similarity data set, for three different combination of links. The figure shows that using mostly hyperlinks shortens the average path length that is used for the computation, while using more content links lengthens the path lengths.

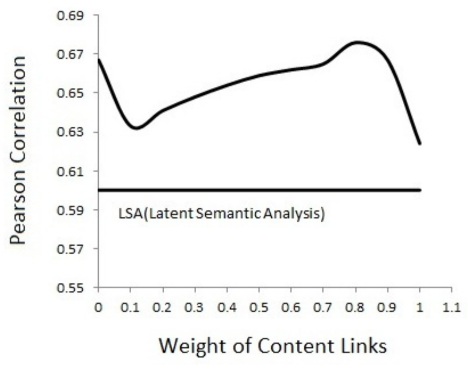


Figure 4.8: Pearson correlation coefficient r between *VP* results and human judgments on document similarity, depending on the weight of the hyperlinks in the combination (the weight of the content links is the complement to 1). The best score of $r = 0.676$ is reached when hyperlinks have less weight than content links, but are still used. The result of LSA, $r = 0.60$, is quoted from Lee et al. [2005].

4.8 Document Clustering

This section describes the experimental setting and the results of applying the text relatedness measure defined above to the problem of document clustering over the 20 Newsgroups dataset.[6] The dataset contains about 20,000 postings to 20 news groups, hence 20 document classes with about 1,000 documents per class. We aim here at finding these classes automatically, using for testing the entire data set without using any part of it as a training set. The knowledge of our system comes entirely from the Wikipedia network and the techniques described in Sections 4.3–4.4.1 for computing distances between two texts projected onto the network. We also experimented with the embeddings trained over *VP* similarities described in Section 4.4.2.

4.8.1 Setup of Experiment on 20 Newsgroups

We first compute a similarity matrix for the entire 20 Newsgroups data set, with the relatedness score between any two documents being VP^T. For tractability, we fixed $T = 10$, a value that gives sufficient precision, as studied empirically in Section 4.5 above. Similarly, we empirically set the absorption probability of the random walk at $1 - \alpha = 0.2$. Based on α and T, the results in Section 3.2 allowed us to compute the error bound of the truncation. So, the choice of α and T was also guided by the fact that for a smaller α, fewer steps (T) are needed to achieve the

[6]The dataset is distributed at http://www.cs.cmu.edu/afs/cs.cmu.edu/project/theo-20/www/data/news20.html, see also [Mitchell, 1997, Chapter 6].

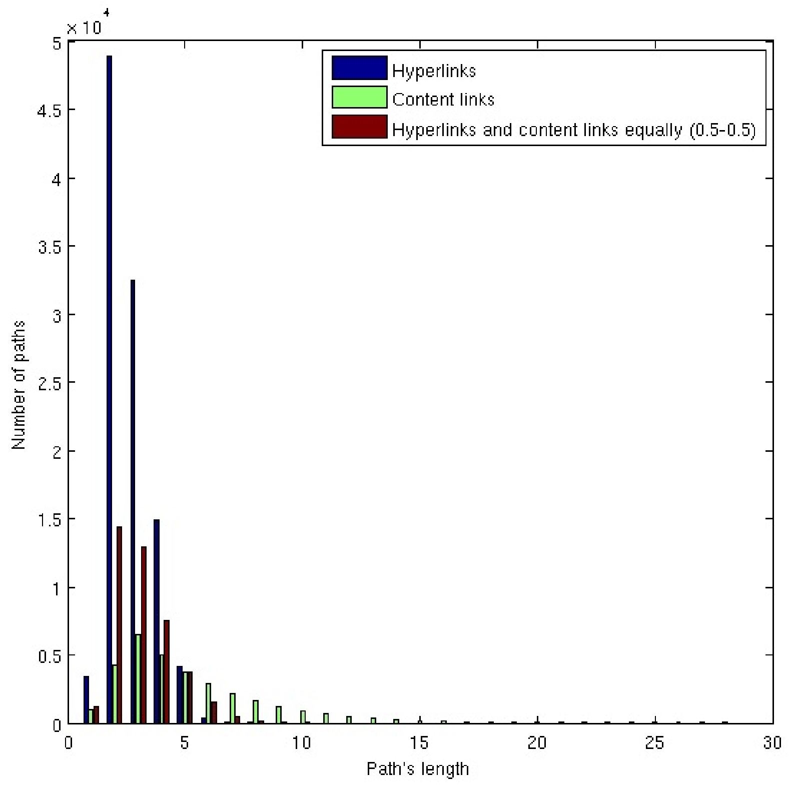


Figure 4.9: Frequencies of the path lengths contributing to *VP*, averaged over the document similarity data set, in three configurations of the system (left to right for each integer value): hyperlinks only, content links only, and equal weights for both types.

same approximation precision because of the penalty set to longer paths. In this application, instead of computing VP^T between all possible pairs separately, we filled one row of the matrix at a time using the approximations we proposed. To use the embeddings we simply transform all the documents by the embeddings matrices and then run the clustering algorithm over the transformed documents in the projection space. We trained two matrices, A and B, for each Wikipedia graph. The qualitative behavior of the results by using A and B is very similar. Here we report only the results of the transformation by A to help readability of the results.

Clustering is performed using a k-means algorithm over each of the similarity matrices and feature representations. The similarity metric between two representation vectors is cosine similarity. Given the randomized initialization of the algorithm, the final clusterings are different in each run of the algorithm.

The quality of the clustering is measured using the Rand Index (RI), which counts the proportion of pairs of documents that are similarly grouped, i.e. either in the same, or in different clusters, in the reference vs. candidate clusterings. RI tends to penalize the smaller clusters, for example if the algorithm clusters together two classes and splits another class, it will receive a high penalty from RI. This happens often in 20 Newsgroups data set, given that some classes are very similar and some other classes are more general and potentially suitable for splitting. As a consequence, the RI values vary largely from one run to another run, making significance testing quite difficult. To gain more information about the quality of the clustering, we consider precision, recall and F-score.

Distance Metric	**Precision**	**Recall**	**F-score**	**RI**
Cosine similarity of TF-IDF vectors	9.29	19.04	12.49	86.67
LSA	19.10	20.64	19.84	91.67
VP on content links	**24.13**	**37.15**	**29.13**	90.94
VP on hyperlinks	18.36	**39.22**	24.72	*87.78*
VP_{Comb} (0.6 content links)	**24.26**	**36.91**	**29.23**	91.03
Embedding from content links (ECL)	21.30	**26.13**	**23.47**	91.48
Embedding from hyperlinks (EHL)	**22.63**	**28.40**	**25.17**	91.56
ECL with regularization	**22.74**	**27.67**	**24.95**	91.68
EHL with regularization	**24.08**	**29.64**	**26.56**	91.80
LDA Blei et al. [2003]	31.00	44.00	36.00	92.00

Table 4.4: Rand Index (RI), precision, recall and F-score of different clustering methods. VP_{Comb} stands for *VP* over a combination of content links (weighted 0.6) and hyperlinks (0.4). Scores in bold are significantly higher than LSA, the one in italics significantly lower (t-test, $p < 0.001$).

Table 4.4 shows average results (over ten runs) of the clustering algorithm using various relatedness measures. The random walk model significantly outperforms the baseline cosine similarity between TF-IDF vectors for document clustering, and it also outperforms LSA in terms of F-score, with most of the relatedness measures. The RI scores do not allow conclusions as their variation does not allow significance judgments.

We observe that the results are lower than those obtained using LDA topic models [Blei et al., 2003][7], but direct comparison between the scores is not wholly informative. LDA was trained on the data set whereas *VP* similarities are ignorant about the data set, so considering this fact, the difference in results appears to be more acceptable. Moreover, the embeddings can be used in other machine learning algorithms to adapt for a specific data set.

The comparison with previously published work is uneasy, as no systematic comparison of clustering methods on the 20 Newsgroups datasets is known to us. In a recent paper, Hu et al. [2009] found an F-score of 14.8% for a baseline word vector method, improved to 19.6% by their use of Wikipedia, for agglomerative clustering. However, for partitional clustering with K-means, both scores increased more than twice (to 38.2% for the baseline and 41.8% for their best method), a result that could not be confirmed independently.

For our approach, the combination of hyperlinks and content links improves the results over using either of them alone. Using the embeddings of the content links reduces the

[7]The results are obtained using the DCA implementation available at http://www.nicta.com.au/people/buntinew/discrete_component_analysis as part of Kei Xing semester project's work.

performance in comparison to the computation of the *VP* values over the graph. On the contrary, using embeddings of the hyperlinks improves the results in comparison to using the *VP* values over the hyperlinks graph.

The embedding learned on *VP* similarities over the hyperlinks appears to provide a more general similarity measure, with does not overfit to the Hyperlinks graph of Wikipedia. The high recall it obtains is related to the larger extension of paths computed with hyperlinks, which can connect many documents together and attach them to the same class, while the high precision obtained using content links is due to their tendency to cluster into smaller neighborhoods.

Although the regularization imposed on the embeddings reduced their predictive power for the *VP* similarities, but it improves the performance on this task.

Computation time using embeddings is (as expected) greatly reduced, as computations are performed in the low-dimensional latent space. Moreover, other unsupervised clustering algorithms can be applied to the documents transformed by the embeddings, e.g. the state-of-the-art clustering algorithm proposed by Bordogna and Pasi [2009].

4.8.2 Comparison of *VP* and Cosine Similarity

To find out in which cases the proposed method improves over a simple cosine similarity measure, we considered a linear combination of the cosine similarity and VP_{Comb} (*VP* over content links weighed 0.6 and hyperlinks weighed 0.4), namely $w \times VP_{Comb} + (1 - w) \times LS$, and varied the weight w from 0 to 1. Considering the k-nearest neighbors of every document according to this combined similarity, we define k-purity as the number of documents with the correct label over the total number of documents k in the computed neighborhood. The variation of k-purity with w, for several sample values of k, is shown in Figure 4.10.

The best purity appears to be obtained for a combination of the two methods, for all values of k that were tested. This shows that VP_{Comb} brings valuable additional information about document relatedness that cannot be found in *LS* only. Furthermore, when the size of the examined neighborhood k increases (lower curves in Figure 4.10), the effect of VP_{Comb} becomes more important, i.e. its weight in the optimal combination increases. For very small neighborhoods, *LS* is almost sufficient to ensure optimal purity, but for larger ones ($k = 10$ or 15), VP_{Comb} used alone ($w = 1$) outperforms *LS* used alone ($w = 0$). Their optimal combination leads to scores that are higher than those obtained for each of them used separately, and, as noted, the weight of VP_{Comb} in the optimal combination increases for larger neighborhoods.

These results can be explained as follows. For very small neighborhoods, the cosine lexical similarity score with the nearest 1–5 documents is very high, as they have many words in common, so *LS* is a good measure of text relatedness. However, when looking at larger neighborhoods, for which relatedness is less based on identical words, then VP_{Comb} becomes more effective, and *LS* performs poorly. Therefore, we can predict that VP_{Comb} will be most

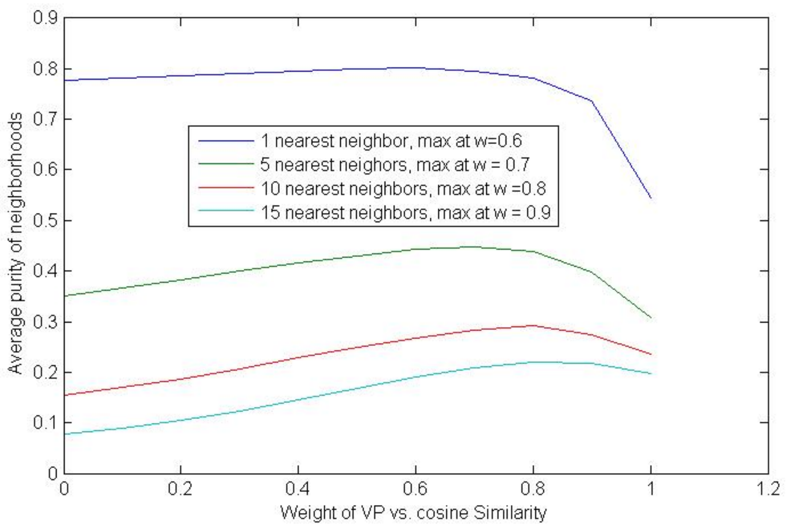


Figure 4.10: Values of k-purity (vertical axis) averaged over all documents, for neighborhoods of different sizes k. The horizontal axis indicates the weight w of visiting probability vs. cosine lexical similarity in the formula: $w \times VP_{Comb} + (1 - w) \times LS$.

relevant when looking for larger neighborhoods, or in order to increase recall. VP_{Comb} should also be relevant when there is low diversity among document words, for instance when all documents are very short.

4.9 Text Classification

We showed in the previous section that using the *VP* similarities over Wikipedia hyperlinks and content links graphs improved text clustering. Although text clustering is an important application, there have been more studies on text classification, i.e. when labeled examples are available for learning. In this section, we investigate this problem by using the embeddings that were learned over *VP* similarities in Section 4.4.2 above. Embeddings can easily be integrated with any distance learning algorithm as prior knowledge or as an initial state. We thus designed a distance learning algorithm for this purpose, which we compared (using various embeddings) to an SVM text classifier, outperforming its score when few training examples are available.

4.9.1 Distance Learning Classifier

We built a distance learning classifier which learns, given a training set, a similarity measure so that for each data point in the training set, its similarity to data points with the same label (or class) is higher than data points with different labels. This classifier is essentially very similar to a large margin nearest neighbor classifier Weinberger et al. [2006], with some changes that make it applicable to large scale text classification problems with a large number of features (here, words).

We define the similarity between two documents i and j represented by TF-IDF vectors x_i and x_j as $x_i AA' x'_j$, where A is a matrix $n \times m$, n being the size of the feature dictionary and m size of the latent space. If $\mathscr{C}(i)$ denotes the class (label) of document i, then we define the following loss function L over the training set:

$$L = \sum_{i} \sum_{\mathscr{C}(j)=\mathscr{C}(i)} \sum_{\mathscr{C}(z)\neq\mathscr{C}(i)} max(0, M - x_i AA' x'_j + x_i AA' x'_z)$$

M is a margin which is set to 0.2 in our experiments. We performed the optimization of L by stochastic gradient descent. At testing time, we proceeded similarly to the k-nearest neighbors algorithm: we chose the k closest documents from the training set according to the learned distance and then returned the class (label) resulting from the majority vote.

Our goal here is to show that starting from the prior knowledge obtained from *VP* similarities over Wikipedia graphs in the forms of the embeddings can improve the performance of the classification, especially when a small number of training samples is available.

4.9.2 20 Newsgroups Classification

We applied the above method to classify texts from the 20 Newsgroups data set. We compared the results of the distance learning algorithm, with various initial points, to a linear SVM method (LIBSVM implementation [Chang and Lin, 2011]), which is a state-of-the-art text classifier. The classification accuracy is given in Table 4.5 for various sizes of the training set. We have trained two matrices over *VP* similarities, A and B, because *VP* similarity is not symmetric. We have experimented with initialization by either A or B, which gives similar results, therefore we show only the results using matrix A. The distance learning classifier with random initialization of the parameters performed poorly, so it is not reported here.

Method	**Size of the training set**						
	40	**100**	**200**	**500**	**800**	**1000**	**1500**
DL + CL embedding	*21.13*	*32.93*	*42.79*	56.57	62.54	65.90	70.57
DL + CL emb. + REG	*22.54*	*34.14*	*44.32*	*57.12*	63.82	66.43	71.10
DL + HL embedding	*21.40*	*34.31*	*44.09*	*57.18*	63.09	66.31	70.65
DL + HL emb. + REG	*22.50*	*35.29*	*46.17*	*58.64*	64.08	66.84	71.14
SVM	7.90	15.93	28.48	52.40	61.76	65.67	70.83

Table 4.5: Classification accuracy for different sizes of the training set over the 20 Newsgroups data set. The accuracy is the average of 10 times run on a randomly divided data set. 'DL' is distance learning classifier, 'CL' is the embedding learned over the content links graph and 'HL' the embedding learned over hyperlinks, 'REG' stands for regularization of the matrix. The numbers in italics are significantly better than the accuracy of the SVM (t-test, $p < 0.001$)

.

The first important observation is that, when the training set is small, the distance learning classifier initialized with the embeddings from *VP* similarities over Wikipedia graphs outperforms the baseline SVM classifier significantly. By adding more and more labeled data, the importance of prior knowledge appears to decrease, presumably because the distance learning algorithm can infer reliable decisions based only on the training data. A similar effect was shown for a method based on deep learning by Ranzato and Szummer [2008], with their method and a TF-IDF/SVM method both reaching 65% accuracy for more than 50 samples per class (corresponding to 1000 total samples) in the training data.

The second observation is that the orthonormality regularization that we imposed on the embeddings again improved the performance. The generalization ability was improved at the price of decreasing slightly the precision in the approximation of the *VP* values. A third observation is that the accuracy using hyperlinks was slightly higher than using content links.

4.10 Information Retrieval

In this section, we apply the proposed distance to information retrieval data from TREC-7 and TREC-8 [Voorhees and Harman, 1999]. The application of our method to a large scale information retrieval task requires the computation of *VP* between a query and the representation of *all the documents* in the repository. By using the approximations proposed in Section 3.2, we can compute T-truncated *VP* between every query and documents in an acceptable amount of time. Firstly, we map all documents in the data set to the Wikipedia graph; then, at query time, each query is mapped to the graph; finally, for each query, VP^T is computed to and from all documents in the data set by using the proposed approximations. In this section, we use $\alpha = 0.8$, $T = 10$ and a combination of hyperlinks and content links in *VP* weighted 0.2/0.8, following observations from previous sections. The time-consuming operation is the first one – viz., mapping a large collection of documents to the Wikipedia graph – but this is done only once, before query time.

We used the TREC-7 and TREC-8 *Ad-hoc Test Collections*.[8] The data set includes a repository of 530,000 documents and two sets of 50 queries. For each query, the data set also provides relevance judgments for a subset of documents considered to be related to the query, as they were retrieved by a pool of search methods, a method that is intended to maximize recall. To study the effect of *VP* in comparison to lexical similarity and TF-IDF scoring in a single retrieval operation, we compute for each document and query a linear combination of VP^T with weight w and of lexical similarity with weight $1 - w$. The weight w thus sets the relative importance of conceptual relatedness vs. lexical similarity, and will serve to illustrate their respective contributions. We will refer to this as the *Combined* similarity. We measure the IR performance through the average precision of the first 10 returned documents (10 is the

[8]These are available at http://trec.nist.gov, and we used more specifically the test queries (topics) numbers 351–400 and 401–450, with the associated relevance judgments. The documents are available from the Linguistic Data Consortium.

typical size of the first result page of Web search engines).

We computed all the scores for $w = 0$ to $w = 1$ with increments of 0.1, and found out that the highest score was reached for $w = 0.1$ for the TREC-7 data set. This optimized value was then tested over the TREC-8 query set (50 topics), leading to an average precision for *Combined* of 0.515 , which improves (+15.4 %) over the precision of lexical similarity alone ($w = 0$), which is 0.446. Egozi et al. [2011] reported precision at 10 on TREC-8 data for various bag-of-words information retrieval systems (Xapian, Okapi, LM-KL-DIR) along with improved results (up to 14%) obtained by using a new retrieval algorithm, which integrates ESA [Gabrilovich and Markovitch, 2009] semantic similarity to the previous systems.[9] Here, we reported the mean average precision at 10, which is the lower bound of precision at 10, but is more informative than it. Direct comparison between our scores and the scores reported in [Egozi et al., 2011] is not possible, as they used different base retrieval systems.

We also examined the precision score for every query separately. In particular, we counted the proportion of queries for which the *Combined* similarity returned more relevant documents than lexical similarity alone. *Combined* similarity outperformed the lexical one on 14 queries, while the reverse was true for 5 queries only, and the scores were identical on the remaining 31 queries. The average score difference between *Combined* and lexical similarity is 0.018 (maximum is 1). This value is not significantly above 0 at the usual levels (e.g. $p < 0.01$), but we found using a normal distribution that the probability of the true difference being zero, given the observations, is only $p = 0.11$. While this does not ensure that the *Combined* similarity is “significantly” better than the lexical one, it still provide encouraging evidence for the utility of *VP* on the TREC-8 test set.

The precision scores of both methods vary considerably across queries. We therefore examined separately the “difficult” queries, defined here as the queries on which lexical similarity had a score of 0.3 or less (meaning it returned between 0 and 3 relevant documents). There are 21 such queries out of 50 on the TREC-8 test set. Over these queries, the *Combined* similarity outperforms the lexical one on 7, while the reverse is true for only one, and 13 queries are a tie. Of course, it might seem unsurprising to see this difference as these queries are “difficult” for the lexical similarity by definition. However, when examining the 20 queries that are “difficult” for the *Combined* similarity, this measure still outperforms the lexical one on 5, while the reverse is true for only two, and 13 are a tie. These results show that *VP* provides complementary information that can improve the results of lexical similarity for IR, especially for queries on which lexical similarity performs less well.

4.11 Learning to Rank by Using *VP* Similarities

We have seen in the previous section that the linear combination of the *VP* and lexical similarities improved only slightly the retrieval results, although *VP* provides additional information.

[9] Precision at 10 was improved as follows: for Xapian from 0.472 to 0.478 (+1.3%), for Okapi from 0.488 to 0.522 (+7.0%), and for LM-KL-DIR from 0.442 to 0.506 (+14.4%).

This motivated us to integrate the *VP* similarity to a learning to rank system [Li, 2011], instead of a linear combination with lexical similarity. We have chosen an approach similar to the discriminative projection learning algorithm introduced by Yih et al. [2011] which exhibits good performance. We have reimplemented the algorithm for the experiments in this section, and we first describe it briefly, then discuss the results.

Assume that for a query q, a document d_r is relevant and d_{nr} is not relevant. The algorithm learns a projection A from TF-IDF vectors of the articles to a latent space such that the similarity between the projections of q and d_r is higher than the similarity between those of q and d_r. Given a training set consisting of (q_i, d_{r_i}, d_{nr_i}), the algorithm minimizes the following loss function L over the training set:

$$L = \sum_i max(0, M - q_i AA' d'_{r_i} + q_i AA' d'_{nr_i})$$

We will show that using the embeddings learned from *VP* as a starting point for minimizing L can help to improve the ranking performance.

To build the training and test sets, we used the 50 queries from TREC-8 and considered for each query the documents labeled as relevant (4728 documents), while unlabeled documents were considered as irrelevant. We divided evenly and randomly the pairs of queries and relevant documents into a training and a test set. The possible number of triples in the training set is very large due to the large number of irrelevant documents. We perform stochastic gradient descent over the training set by choosing at each iteration a query and a relevant document, with an irrelevant document chosen randomly from the rest of the documents. We stop when the training error is lower than a fixed threshold. To test the performance, we report average precision at 10. This is computed over the number of relevant documents that are not used for training. Therefore, when using a larger training set, fewer documents are left for testing and the precision at 10 scores necessarily decrease; however, our interest is in comparing scores for different methods on the same training set.

Method	**Size of the training set**				
	500	**1000**	**1500**	**2000**	**3000**
RL + CL embeddings	13.62	14.40	12.62	11.24	5.54
RL + CL emb. + REG	**17.48**	**17.84**	**14.02**	12.08	5.56
RL + HL embeddings	15.74	15.98	13.10	11.28	5.36
RL + HL emb. + REG	**19.44**	**18.60**	**14.34**	12.46	5.70
RL + random start	16.3	15.16	13.56	12.48	5.34
Cosine between TF-IDF vectors	**21.16**	14.74	9.62	7.98	2.70

Table 4.6: Precision at 10 for different ranking algorithms when training size varies. As in Table 4.5, ‘RL’ is the ranking learner, ‘CL’ is the embedding learned over the content links graph and ‘HL’ the embedding learned over hyperlinks, and ‘REG’ stands for regularization of the matrix.

Table 4.6 shows the precision at 10 of various algorithms by using different initial states and different number of training samples. The first observation is that only when the training size is very small, 10 documents on average for each query, the cosine similarity outperforms the learning to rank algorithms. Otherwise, the performance of the learning to rank algorithms is always better than the cosine similarity.

The second result is that when the number of training examples is small, the learning algorithm initialized with the regularized embeddings outperforms the random initialization. Gradually, when adding more training examples, it becomes less useful to leverage the prior knowledge, as the learning algorithm can solve the problem better simply by looking at training samples. Similarly to the classification experiments, the embeddings learned over the hyperlinks are more useful than the ones learned over the content links.

4.12 Application to Just-in-Time Retrieval

The Automatic Content Linking Device (ACLD) is a just-in-time document retrieval system which monitors an ongoing conversation or a monologue and enriches it with potentially related documents, including multimedia ones, from local repositories or from the Internet [Popescu-Belis et al., 2008, 2011]. The documents are found using a similarity measure between documents and the words obtained from automatic speech recognition (ASR). Results are displayed in real time to meeting participants, or to users watching a recorded lecture or conversation.

ACLD can be performed over live or recorded lectures, for instance in a computer-assisted learning environment for individual students. The ACLD enriches the lectures with related material drawn from various repositories, through a search process that can be guided in real time by its user.

The goal of our method is to use the proposed semantic similarity to improve the relevance of the retrieved documents, and to make the mechanism more robust to noise from the ASR. The goal is to enrich the the conversation with the most related articles from Wikipedia. Query object is built by using the words obtained from ASR in a given timeframe, stop words removed. We map the query to the corresponding articles in Wikipedia network using the method explained in Section 4.4.1. The set of K closest articles based on the symmetric *VP* similarity is returned using the algorithms explained in Chapter 3.

We compared the output of semantic similarity search with that of keyword-based search (see also [Habibi and Popescu-Belis, 2012] for an additional comparison). The ASR transcript of one AMI meeting (ES2008d) was passed to both search methods, and “evaluation snippets” containing the manual transcript for one-minute excerpts, accompanied by the 8-best Wikipedia articles found by each method were produced. Overall, 36 snippets were generated. The manual transcript shown to subjects was enriched with punctuation and speakers’ names, and the names of the Wikipedia pages were placed on each side of the transcript frame.

Subjects were then asked to read each snippet, and decide which of the two document sets was the most relevant to the discussion taking place, i.e. the most useful as a suggestion to the participants. They could also answer 'none', and could consult the result if necessary. Results were obtained from 8 subjects, each seeing 9 snippets out of 36. Every snippet was thus seen by two subjects. The subjects agreed on 23 (64%) snippets and disagreed on 13 (36%).

In fact, the number of true disagreements not including the answer 'none' was only 7 out of 36. Over the 23 snippets on which subjects agreed, the result of semantic search was judged more relevant than that of keyword search for 19 snippets (53% of the total), and the reverse for 4 snippets only (11%). Alternatively, if one counts the votes cast by subjects in favor of each system, regardless of agreement, then semantic search received 72% of the votes and keyword-based only 28%. These numbers show that semantic search quite clearly improves relevance in comparison to keyword-based one.

The query objects built on words obtained from ASR of a meeting fragment are noisy. This noise comes mainly from two sources, first noises entered by ASR error and second, the unfocused nature of the discussions. For example, if a meeting fragment is about reinforcement learning, we might have words like "enforcement" in the query object which results from ASR error. Also there might be words about "game design" as it was mentioned as an application of reinforcement learning. Moreover, the query objects might be consisting of ambiguous words.

In all these cases, when the query is mapped to the corresponding concepts, the effect of noise – which is less consistent with the majority of the query – reduces. In our hypothetical example the majority of the mapped concepts are related to reinforcement learning with high weights (probability) and there are few unrelated concepts with low probabilities. When the random walk starts the related pages, which have high initial weights or connected through either type of links to the high weighted pages, get to the top of the list, and unrelated pages drop to the bottom of the list. Therefore, the second round of denoising of the recommended set is done by the random walk and therefore, the relevance of the recommended set is improved in comparison to the keyword based approach.

4.13 Perspectives: Two-Stage Random Walks

In addition to the above experiments, we examined two-stage random walks, in which at the first stage, the network is built using one set of weights on the links, and in the second stage using a different set. The hypothesis here is that some links might be more useful when explored first, while some others might be more useful when explored later, as discussed by Collins-Thompson and Callan [2005].

For the word similarity task, we focused on ε-truncated two-stage walks in which one combination of links is used for the first three steps of the random walker, and a different combination from the fourth to the last steps. The choice of three steps was empirical, by looking at the average length of single-stage ε-truncated walks. We report the ρ scores of three significant combinations of weights for this scenario in Table 4.7, including the best we found, which reached $\rho = 0.714$, higher than the one-stage walks (see Section 4.6). As the optimization took place on the test data, this is not a competitive score, but intends to show that two-stage walks are a promising approach, in particular exploring the hyperlinks first and then the content links.

A similar analysis to the one shown in Figure 4.7 explains why scores improve in the two-stage random walk in Table 4.7, which travels hyperlinks first (thus expanding the possible paths), and then content links (following precise neighborhoods). In the case of exploring hyperlinks first and content links second, there are longer paths in comparison to using only hyperlinks. In the case of exploring hyperlinks in the second stage, there are many long paths in comparison to other scenarios.

(Hyperlink weight, Content link weight)	ρ
(1.0, 0.0) for 3 steps, then (0.0, 1.0)	.684
(0.0, 1.0) for 3 steps, then (1.0, 0.0)	.652
(0.7, 0.3) for 3 steps, then (0.0, 1.0)	**.714**

Table 4.7: Spearman rank correlation coefficient ρ between automatic and human word similarity when two-stage random walk is used for *VP*. In parentheses the respective weights of hyperlinks and content links. The best result, $\rho = 0.714$ is found when both hyperlinks and content links are used for the first three stages (with weights 0.7 vs. 0.3), but only content links are used for latter stages.

The results of VP following two-stage random walks with several meaningful combinations of links are given in Table 4.8 for document similarity data set. The scores on document similarity can be slightly improved to $r = 0.680$ if hyperlinks are mostly explored in the first steps, and then only content links are followed which is congruent with our finding about two stage random walk for word similarities.

(Hyperlink weight, Content link weight)	r
(1.0, 0.0) for 3 steps, then (0.0, 1.0)	.667
(0.0, 1.0) for 3 steps, then (1.0, 0.0)	.635
(0.8, 0.2) for 3 steps, then (0.0, 1.0)	**.680**

Table 4.8: Pearson correlation coefficient r between two-stage *VP* results and human judgments on document similarity. The best result (0.680) is found when for the first three stages both hyperlinks (0.8) and content links (0.2) are used, but only content links are used for latter stages.

5 Joint Similarity Learning for Predicting Links in Multi-Link Networks

In this chapter, we address the problem of link prediction on large networks with links of multiple types (or in short multi-link networks) by proposing two joint similarity learning architectures over the nodes' attributes. The first model is a similarity metric that consists of two parts: a general part, which is shared between all link types, and a specific part, which learns the similarity for each link type specifically.

The second model consists of two layers: the first one, which is shared between all link types, embeds the objects of the network into a new space, while the second one learns the similarity between objects for each link type in this new space. The similarity metrics are optimized using a large-margin optimization criterion in which connected objects should be closer than non-connected ones by a certain margin.

A stochastic training algorithm is proposed, which makes the training applicable to large networks with high-dimensional feature spaces. The models are tested on link prediction for two data sets with two types of links each: TED talks (1150 items) and Amazon products (10,000 items). The experiments show that jointly modeling the links in our framework improves link prediction performance significantly for each link type. The improvement is particularly high when there are fewer links available from one link type in the network. Moreover, we show that transfer learning from one link type to another one is possible using the above frameworks.

The chapter is organized as follows. Following an outline of the motivations, we introduce in Section 5.2 the joint models for multiple-type links, first by defining the shared similarity model, and then by defining the -layer similarity model. We then describe the large-margin method for training the similarity functions (5.2.4). We also show how to generalize the proposed framework to jointly model link prediction and classification in a network (5.2.5). The experiments described in Section 5.3 on the above-mentioned networks demonstrate the predictive power of the joint model, in comparison with separate models, as well as with SVM Rank and cosine similarity.

5.1 Motivations

The problem of new link prediction in networks is particularly relevant to networks that are collaboratively built over time, whether they are document-based (such as Wikipedia, or databases of products or multimedia records) or person-based (social networks such as Facebook or LinkedIn) or both. In such situations, some nodes become connected over time due to similarities or affinities that are noticed and validated by users. Predicting such connections is a functionality which is valuable to speed up network construction, for instance by presenting these connections as recommendations to users, as discussed in the introduction to this thesis. However, this is particularly challenging when links are of multiple types. Moreover, accurate modeling of the connections in the network can be useful to predict the evolution of the network or even to perform marketing via the network.

We introduce in this chapter two joint link prediction models for networks with multiple link types. Link prediction is formulated as a learning to rank problem: given a query node, all other nodes must be sorted according to a score that is related to the likelihood of creating a link between them and the query node. The proposed link prediction models learn a similarity metric according to which connected nodes are closer than non-connected nodes.

Two different joint models will be proposed. The *shared similarity model* consists of two parts: the general part, which is a shared similarity function between all types of links, and the specific part, which learns similarity specifically for each type of links. The *two-layer similarity model* consists of two layers: in the first layer, objects in the network are embedded into a new space, while the second layer learns the similarity between objects for each link type specifically in this new space. The first layer is shared between all link types and can also be considered as a representation of objects which is common across all link types.

5.2 Joint Similarity for Multiple-Type Link Prediction

Link prediction can be viewed as a ranking problem in which all objects in the network are ranked based on their similarity score with respect to a query object. A "better" ranking is one that places objects that are actually linked to the query object at the top of the ranked list. The score between the two objects in the network can be considered as a similarity metric (or, conversely, a distance one), and link prediction can be interpreted as a task in which linked objects should be closer to the query object, in comparison to the non-linked objects.

We model the link structure of a network by learning a similarity measure which, for each object in the network, assigns larger scores to the objects that are linked to it than to those that are not. To model N different types of links, we train N different similarity functions, so that we model the link structure of each type separately. However, in many real-world networks, links of a certain type are not entirely independent from links of other types. To consider this dependency, each link-type similarity model shares information with the other models. We will show in Section 5.3 that the joint modeling increases generalization abilities, hence link prediction performance, especially when there are few links from one specific link type.

5.2.1 Similarity Learning Framework

In this section, we build a similarity based ranker to model links in the network. The ranker returns a score for a given query object q and a target object t by using a similarity function over their features. If $f_i(q,t)$ represents the ranker for link type i, and x_o is the feature vector of an object o from the network, then we have:

$$f_i(q,t) = similarity_i(x_q, x_t)$$

The $similarity_i$ function computes a similarity for link type i between objects q and t using their attributes, following a classical approach for learning to rank methods. Many similarity functions (or, equivalently, distance metrics) with various learning abilities have been studied in the literature, for instance RankNet Burges et al. [2005], polynomial semantic indexing Bai et al. [2009] or structure preserving metric learning Shaw et al. [2011] (see also Chapter 2 above). In this study, we use inspiration from these previous studies and define the similarity function as follows:

$$similarity_i(x_q, x_t) = x_q \times M_i \times x_t'$$

where matrix M_i is a $Z \times Z$ matrix, Z being the size of the features, and x_t' is the transpose of x_t. The $similarity_i$ function is not necessarily symmetric, which makes it compatible with networks with directed links. In practice, to make training and storage possible, particularly when dealing with high-dimensional data such as text in a word vector representation, a low-rank factorization of M is considered for training Bai et al. [2009], Shaw et al. [2011]. Each ranker in the above formulation is ignorant about the other link types, therefore we call this model separate model and it is used in the experiments section as a baseline. We introduce two models by extending this approach to share information among rankers.

5.2.2 Shared Similarity Model

To introduce joint modeling among rankers, and share information between them, we assume that each matrix M_i consists of two parts: a general matrix noted G, which is shared between all rankers, and a specific matrix noted S_i, which is learned separately for each link type i. Hence, $M_i = G + S_i$. The similarity function can thus be formulated as:

$$similarity_i(x_q, x_t) = x_q \times (G + S_i) \times x_t' = \underbrace{x_q \times G \times x_t'}_{\text{General Similarity}} + \underbrace{x_q \times S_i \times x_t'}_{\text{Specific similarity for the type } i}$$

The similarity function thus in its turn consists of two parts: first, a general similarity measure which is shared (and identical) across link types and second, the specific similarity measure which is adapted to each link type. Matrices G and S_i s are the parameters of the similarity function which are going to be trained.

Matrix G would represent the correlation between link types. To better explain the role of G let us consider a network with two link types and extreme hypothetical cases: when the two link types are independent, and when they are identical. If the two link types are independent, then the matrix G will be the 0 matrix and the model would be equivalent to the separate model introduced in the previous section. On the other hand, if the two link types are identical, then matrix G is identical to the link specific matrices S_i. For the other situations between these two extreme cases, matrix G would be trained to represent the correlation between the two link types.

However, this model is able to handle dependencies between the link types only if they are positively correlated. In cases where two link types in the network are negatively correlated, this correlation would not be modeled by G since having a link of the first type between two objects prevents us from having the other type of link between them and G would be simply 0. For example consider a network consisting of researchers and two types of collaboration between them: first intra-institute collaboration and second, inter-institute collaboration. If there is a link of the first type between two researchers in the network, then the second link type can not exist between them. This model can not represent this negative correlation between link types and is equivalent to the separate model on this network.

The above-mentioned problem is the main drawback of our first model. In the following section we introduce a two-layer similarity model to overcomes this problem.

5.2.3 Two-layer Similarity

The two-layer similarity model consists of two layers: the first layer embeds objects in the network to a new space, and then the similarity for each link type is learned specifically in this new space. The first layer which embeds the objects to a new space is the same for all link types. The first layer can be viewed as a *shared representation* for objects among all link types. Figure 5.1 shows schematically the two-layer similarity model.

The matrix $A_{Z\times K}$ transforms the objects to a new space with dimension K where Z is the dimension of objects' features and matrix M_i is the matrix representing the similarity for the link type i in the new space. The similarity function can thus be formulated as:

$$similarity_i(x_q, x_t) = \underbrace{(x_q \times A)}_{\text{Shared representation}} \times (M_i) \times \underbrace{(x_t \times A)'}_{\text{Shared representation}}$$

Matrices A and M_i are the parameters of the model which are going to be trained. This two-layer model overcomes the main problem of the previous model, since it can also model

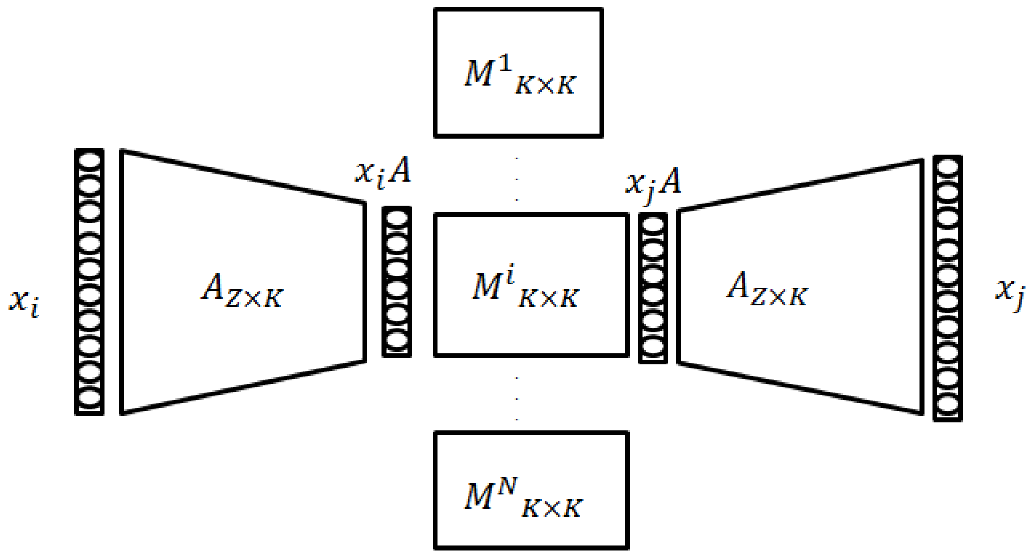


Figure 5.1: A schematic representation of the two-layer similarity model. Matrix A embeds the objects in the network to a K-dimensional space. Matrix M^i computes the similarity between objects for link type i.

negatively correlated link types through the shared representation. To explain intuitively how this can be done assume again the hypothetical network of researchers and the two types of relations between them. If we assume there are two institutes in our network, the first layer could potentially embed objects into the two clusters in the latent space corresponding to the two institutes. Then the similarity matrices M_i in the latent spaces would learn that first type similarity is higher between the objects in the same cluster, and second type similarity between objects in different clusters. If there exist a first link type between two researchers, then they are embedded in the same cluster and the score of the second link type drops, therefore the shared representation enables joint modeling of even negatively correlated link types.

5.2.4 Training the Joint Models

We can formulate the learning criterion for a specific link type as a pairwise ranking problem, in which given an object, other objects that are linked in the training data should be ranked higher than non-linked ones. Below, we formalize the learning-to-rank problem and then place the joint rankers introduced in the previous section in this framework.

We make use of a loss function noted $L(\cdot,\cdot)$ to evaluate the prediction result of ranking upon training. The feature vectors are ranked according to their scores, and the resulting ranking is evaluated against the corresponding expected links (known in the training set). If the feature vectors of linked objects are ranked higher, then the loss will be small, otherwise it will be large.

We define here a pairwise hinge loss function L, which attempts to preserve the pairwise order between objects in the training set by maintaining a margin between the scores of each pair Bai et al. [2010]. For instance, in a friendship social network, the hinge function (computed only using feature vectors) aims to preserve the order between the scores of the friend pairs and the non-friend pairs, scoring all friends higher than all non-friend. Similarly, in the case of hyperlinked documents, the score of the linked documents pairs should be higher, with a certain margin, than the score of the non-linked documents.

The goal of training is to approximate the parameters of a similarity function for rankers which minimizes the pairwise hinge loss function L over the network $\mathcal{G}$ with the various types of links that are given in the training set. Training is performed over the graph $\mathcal{G} = (V, E_1 \cup E_2 \cup \ldots E_N)$, where V is the set of vertices (objects) and E_i is the set of edges (links) of type i.

Training the Shared Similarity Model

The empirical risk minimization by using a pairwise hinge loss function over the training set is as follows. We start from training sets for each link type i:

$$Train_i = \{(a,b,c)|(a,b) \in E_i \wedge (a,c) \notin E_i\}$$

The goal is to minimize the loss function L as follows:

$$MinL = \frac{\lambda}{2}||G||_F^2 + \frac{\lambda}{2}\sum_i ||S_i||_F^2 + \frac{\sum_i \sum_{(a,b,c)\in Train_i} max(0, d - f_i(a,b) + f_i(a,c))}{(\sum_i |Train_i|)}$$

The first two terms, the Frobenius norms of matrices G and S_i, enforce a regularization on the parameters and prevent the arbitrary increase of the parameters in the optimization. The third term of the loss function maintains the order, with a margin d, between the scores of the connected nodes and the unconnected nodes for link type i.

If we consider the definition of f_i from the previous section and the decomposition of M_i into G and S_i, then the subgradients of the loss function with respect to G and to S_i are the following:

$$\nabla L(G) = \lambda G + \frac{\sum_i \sum_{f_i(a,b)-f_i(a,c)<d} x_a' \times (x_c - x_b)}{\sum_i |Train_i|}$$

$$\nabla L(S_i) = \lambda S_i + \frac{\sum_{f_i(a,b)-f_i(a,c)<d} x_a' \times (x_c - x_b)}{|Train_i|}$$

If the training graph $\mathcal{G}$ is large, then minimizing the above summation is not tractable. To overcome this problem we make use of a stochastic gradient descent algorithm in which at each iteration we select N samples (a,b,c), one randomly from each $Train_i$ set, and perform gradient descent on each of them. As we are interested in modeling large scale graphs, we will always make use of this stochastic gradient descent algorithm in the rest of the chapter. The subgradients of the approximate loss function at iteration t are the following, for an iteration over one of the samples $(a,b,c) \in Train_i$:

$$\bigtriangledown L_t(G) = \lambda G^t + \{ \begin{array}{rl} x'_a(x_c - x_b) & \text{if } f_i(a,b) - f_i(a,c) < d \\ 0 & \text{otherwise} \end{array}$$

$$\bigtriangledown L_t(S_i) = \lambda {S_i}^t + \{ \begin{array}{rl} x'_a(x_c - x_b) & \text{if } f_i(a,b) - f_i(a,c) < d \\ 0 & \text{otherwise} \end{array}$$

The update of the parameters is done consequently as follows:

$$G^{t+1} = G^t - \eta_t \bigtriangledown L_t(G)$$
$${S_i}^{t+1} = {S_i}^t - \eta_t \bigtriangledown L_t(S_i)$$

where η_t is the learning rate at the iteration t.

Analysis of the training algorithm. The first observation is that the time complexity of each iteration of the stochastic gradient algorithm is $O(N_Z^2)$ where N_Z is the average of the number of non-zero features of the x_a, x_b and x_c. In the textual domain, the number of non-zero features N_Z is usually a couple of hundreds, which is much smaller than the dimension of a word vector representation of the data, which is usually several hundred thousands (number of all possible words).

We can follow the same approach that is used to prove the convergence of the PEGASOS algorithm in Shalev-Shwartz et al. [2011] in order to prove the convergence of the training algorithm presented above. In fact, we can show that the above algorithm is a special case of PEGASOS. Prove of the convergence is at the of this chapter in section 5.4.

Low-Rank Factorization. The dimension of G and S_i is Z^2 where Z is the number of features of the data points. In practice, to make training and storage possible, particularly when we are dealing with high dimension data such as text, a low-rank factorization of G and S_i should be considered. In this case we assume that $G = AA'$ and $S_i = S_{i1}S'_{i2}$ so that A, S_{i1} and S_{i2} are matrices from Z to a lower dimension. Given that M_i is not necessarily symmetric we decompose the specific part S_i to two lower-rank matrices. Also we change the regularization on G and S_i with the regularization on A, S_{i1} and S_{i2} in the objective function. The objective function consisting of the low-rank matrices is not convex any more, but in practice it has been shown that the low-rank factorization performs well Bai et al. [2009]. The subgradients are as following for a given sample $(a, b, c) \in Train_i$:

$$\bigtriangledown L_t(A) = \lambda A^t + \left\{ \begin{array}{ll} x_a'(x_c - x_b)A^t + (x_c - x_b)' x_a A^t & \text{if } f_i(a,b) - f_i(a,c) < d \\ 0 & \text{otherwise} \end{array} \right.$$

$$\bigtriangledown L_t(S_{i1}) = \lambda S_{i1}^t + \left\{ \begin{array}{ll} x_a'(x_c - x_b)S_{i2}^t & \text{if } f_i(a,b) - f_i(a,c) < d \\ 0 & \text{otherwise} \end{array} \right.$$

$$\bigtriangledown L_t(S_{i2}) = \lambda S_{i2}^t + \left\{ \begin{array}{ll} (x_c - x_b)' x_a S_{i1}^t & \text{if } f_i(a,b) - f_i(a,c) < d \\ 0 & \text{otherwise} \end{array} \right.$$

Training two-layer Similarity Model

Similarly to the previous section, for the two-layer model the empirical risk minimization using a pairwise hinge loss function over the training set is as follows. We start from training sets for each link type i:

$$Train_i = \{(a,b,c) | (a,b) \in E_i \wedge (a,c) \notin E_i\}$$

The goal is to minimize the loss function L as follows:

$$MinL = \frac{\lambda}{2}||A||_F^2 + \frac{\lambda}{2}\sum_i ||M_i||_F^2 + \frac{\sum_i \sum_{(a,b,c)\in Train_i} max(0, d - f_i(a,b) + f_i(a,c))}{(\sum_i |Train_i|)}$$

Again, the first two terms enforce a regularization on the parameters and prevent the arbitrary increase of the parameters in the optimization. If we consider the definition of f_i then the subgradients of the loss function with respect to A and to M_i are the following:

$$\bigtriangledown L(A) = \lambda A + \frac{\sum_i \sum_{f_i(a,b) - f_i(a,c) < d} (x_a'((x_c - x_b)A) + (x_c - x_b)'(x_a A)) \times M_i}{\sum_i |Train_i|}$$

$$\bigtriangledown L(M_i) = \lambda M_i + \frac{\sum_{f_i(a,b) - f_i(a,c) < d} (x_a \times A)'((x_c - x_b) \times A)}{|Train_i|}$$

If the training graph $\mathcal{G}$ is large we make use of a stochastic gradient descent algorithm in which at each iteration we select N samples (a, b, c), one randomly from each $Train_i$ set, and perform gradient descent on each of them. The subgradients of the approximate loss function at iteration t are the following, for an iteration over one of the samples $(a, b, c) \in Train_i$:

$$
\begin{aligned}
&marg = f_i(a,b) - f_i(a,c)\\
&\bigtriangledown L_t(A) = \lambda A^t + \\
&\{ \begin{array}{rl} (x'_a((x_c - x_b)A^t) + (x_c - x_b)'(x_a A^t)) \times {M_i}^t & \text{if } marg < d \\ 0 & \text{otherwise} \end{array}\\
&\bigtriangledown L_t(M_i) = \lambda {M_i}^t + \\
&\{ \begin{array}{rl} (x_a \times A^t)'((x_c - x_b) \times A^t) & \text{if } marg < d \\ 0 & \text{otherwise} \end{array}
\end{aligned}
$$

The update of the parameters is done consequently as follows:

$$
\begin{aligned}
&A^{t+1} = A^t - \eta_t \bigtriangledown L_t(A)\\
&{M_i}^{t+1} = {M_i}^t - \eta_t \bigtriangledown L_t(M_i)
\end{aligned}
$$

where η_t is the learning rate at the iteration t.

Analysis of the training algorithm. The time complexity of each iteration of the stochastic gradient algorithm is $O(K^2 + N_Z \times K)$ where N_Z is the average of the number of non-zero features of the x_a, x_b and x_c and K is the size of latent space. The loss function is not convex in the two-layer similarity model and, therefore, the initialization of the parameter can have a significant effect on the performance.

5.2.5 Generalization for Related Problems

In this section we show that the above architectures for joint link prediction on multi-networks can be extended for the similar problems.

Joint Classification and Link Prediction

We show that a joint classification and link prediction task over a network can be reformulated as a multi-link prediction task, thus enhancing the generality of our proposal. Indeed, in most real-world networks, objects are labeled with a certain finite label set, and these labels are not independent from the link structure. For example, in a papers/citations network, it is more likely that machine learning papers cite other machine learning papers rather than biology papers. Conversely, a paper which cites many machine learning papers is more likely to be a machine learning paper rather than a biology paper.

We introduce here a new link type based on the similarity of objects' labels: every two objects in the network that have the same label will be connected by such a link. Therefore, the classification and the link prediction tasks can be casted to a multi-type link prediction problem, in which one of the link types is based on the similarity of objects' labels. Consequently, the goal of learning for this task is to find similarity functions for which objects with the same labels are closer than the other objects, and (as above) linked objects are closer than non-linked objects. For the classification task, i.e. to determine the label of a yet unlabeled object, we consider the majority label of the closest objects to that object.

The empirical risk minimization for joint classification and link prediction by using a pairwise hinge loss function over the training set is as follows. To simplify notations, we assume that there is only one link type i in the network, and we note $\mathscr{C}(o)$ the label or class of an object o. First, the training sets are defined as:

$$Train_L = \{(a,b,c)|(a,b) \in E, (a,c) \notin E\}\ ,\ Train_C = \{(a,b,c)|\mathscr{C}(b) = \mathscr{C}(a), \mathscr{C}(c) \neq \mathscr{C}(a)\}$$

Then, training is defined as minimizing the loss function L :

$$MinL = \lambda \times Regularization\ on\ the\ parameters + \frac{\sum_{(a,b,c)\in Train_L \cup Train_C} max(0, d - f_i(a,b) + f_i(a,c))}{(|Train_C| + |Train_L|)}$$

Both models introduced above can be used as the similarity model in the above formulation. In the shared similarity model case we consider a general similarity function which is shared between classification and link prediction tasks, along with specific similarity functions for each task separately. In the two-layer model the first layer is the shared representation between classification and link prediction task, and the second layer shows the specific similarity in the embedded space. The training algorithm is identical to the case of link prediction on the multi-type links networks, which was discussed above.

Networks without Attributes

Throughout this chapter, we assumed that the similarity functions are learned over node attributes. However, in networks where *nodes do not have attributes* or attributes are not predictive , the proposed framework can still be applied. For each node, a feature vector is defined of the size of all nodes in the graph, in which only the element corresponding to the node has value 1 and the others are set to 0.

In this case, the low rank factorization of the similarity matrix in shared similarity model, or the first layer in the two-layer model is similar to the existing methods for network matrix decomposition, such as singular value decomposition or non-negative matrix factorization, except that our method enforces the large margin criterion and is trained jointly. The empirical exploration of this case is the topic of future work.

Inter-Network transfer learning

We show in this chapter that we can transfer the learning from one link type to another link type through the shared information part. *Transferring learning from one network to another one* is another interesting and related problem. Considering two different online social networks, one based on friendship (such as Facebook) and the second one based on professional relations (such LinkedIn), the model described here could be applied to share the learning between the two networks as long as it is possible to build identical feature vectors for the nodes in both networks. This problem also deserves further investigations.

5.3 Experimental Results

Many real-world networks have multiple types of links. For example, in online social networks, people have different types of relationships and interactions. In many such networks, especially those that are collaboratively built, some objects often have fewer links from one link type. In this section, we consider two networks: TED videos and Amazon products which are explained. We show that our joint link prediction algorithm improves the link prediction performance, especially when there are fewer links from one link type.

5.3.1 Network of TED Talks

TED is an online audio-visual broadcasting platform for the TED talks. [1] We consider a network consisting of nearly 1,200 TED talks, with metadata and transcripts, linked by two types of links. The first type relate two talks based on the similarity of contents. We derive such content links from the suggestions of similar talks made by experts from TED. The second type of links connects two videos if they co-exist in many users favorite lists. Since less popular talks do not appear in many people's favorite lists, there are fewer links based on users favorite lists for such talks, and presumably the links for these talks are not very reliable. Note that recognizing automatically that two talks have similar contents is difficult, but inference about the content can be done more easily when two talks co-occur in many people's favorite lists.

More specifically, the second type of links based on users' favorite lists is built as follows. Each talk is represented by a vector in the space of all users, and an element of the vector is 1 if the talk is in the favorite list of the corresponding user, and 0 otherwise. Given one specific talk, the rest of the talks can be sorted according to cosine similarity between their vectors and the initial talk in the space of users. If two talks co-exist in many users' favorites list, the cosine similarity between their vectors in the space of users is high. Therefore, to build the second type of links, we connect each talk with the top k talks based on cosine similarity between their vectors in the users space. We will call this type of links "co-liked".

[1]This data set has been collected by Nikolaos Pappas from Idiap, who has kindly shared it for the work presented here. See http://www.ted.com for the original website.

For example, the TED talk “Lucien Engelen: Crowdsource your health” has two content links (two recommendations of topically-related talks from TED experts): “Jacqueline Novogratz on patient capitalism” and “Nicholas Christakis: How social networks predict epidemics”. The three highest score co-liked talks are: “Marco Tempest: Augmented reality, techno-magic”, “Aparna Rao: High-tech art (with a sense of humor)” and “Sheikha Al Mayassa: Globalizing the local, localizing the global”. We can observe that the related videos are about the same topic, but co-liked videos are much less predictable from the content.

We build a feature vector for each talk from three sources: (1) speaker name, (2) title of the talk, and (3) talk description as provided by TED (a short text). Each feature vector has a dimension of 4,771 (based on a filtered vocabulary of 4,771 words), but vectors are very sparse, on average with only 35 non-zero coefficients.

5.3.2 Network of Amazon Products

We build a network from a subset of Amazon products (described in Leskovec et al. [2007]) with two types of links between them. The first type comes directly from the Amazon website: for each product, some of the co-purchased products are shown by Amazon. Each product in the network is connected to the products which are claimed to be mostly co-purchased by the Amazon website. We call this type of links “co-purchased”.

Each product is assigned to at least one category, often to more. Categories form a hierarchy, from more general ones to the most detailed ones. We represent each product as a vector in the space of all categories: if the product is assigned to a category, the corresponding element is 1, otherwise it is 0. Cosine similarity between category vectors of two products is high if they are mostly in the same categories from the hierarchy. To build this type of the links, we connect each product to the k most similar products using cosine similarity between the category vectors of the products. We call this type of links “category links”.

For example, consider the book “Writers in the Schools: A Guide to Teaching Creative Writing in the Classroom”. This book is co-purchased with the following other books: “The Magic Pencil: Teaching Children Creative Writing [...]” and “Journal Jumpstarts: Quick Topics and Tips for Journal Writing”. Two category links are “Reading, Writing, and Learning in ESL (2nd Edition)” and “Time to Know Them: A Longitudinal Study of Writing and Learning at the College Level”.

Features of each product in the network come from the title and description of the product. We use a dictionary of 3,765 words and sampled 10,000 products to build our network of products. So, our second network is about eight times larger than the first one.

5.3.3 Prediction Performance

To study the link prediction ability of our models experimentally, we split the objects of each network into disjoint training and test sets. The training is performed on the objects in the training set and the links between them, excluding links from these objects to the test set. To evaluate prediction performance, for each object in the test set, all the objects in the entire data set (training plus test sets) are ranked by the ranker, and the Mean Average Precision (MAP) is calculated for the resulting ranked list according to the links in the test set. The final scores are the average of 5-fold cross validation, with 80% of the data used for training and 20% for testing, in each fold.

We are interested in observing the prediction performance of the joint model when there are few available links from one link type. To perform such an analysis, we make available to the joint learner a full training set for one link type (80% of the objects in the network), and a variable-sized training set for the other link type, varying from a smaller subset (30% of the nodes) to the full training set (80%) by fixed increments (10%). The test set is always fixed (20%). We observe the evolution of prediction performance with the size of the training set for the second type of links.

Table 5.1 shows the average MAP for the objects in the test set for the co-purchased links, in the Amazon products network, for several models. As explained, the size of the co-purchase training set varies from 30% to 80% of the entire network, while the training set for category links is constant (80%). Both 'separate' and shared similarity model are using low rank factorization of M due to the dimension size of the features, with a latent space of dimension 100. The size of latent space in two-layer network is set to 100 as well. All matrices are initialized by random values.

Model	**Proportion of training set**					
	0 (transfer)	**0.3**	**0.4**	**0.5**	**0.6**	**0.8**
Cosine Similarity	8.03	8.03	8.03	8.03	8.03	8.03
SVMrank	–	8.94	9.06	8.99	8.92	9.11
Separate Model	–	8.39	8.53	8.70	8.92	9.13
Shared Similarity	*8.43*	**9.16 (+9.1%)**	**9.21 (+7.9%)**	**9.31 (+7.0%)**	**9.56 (+7.1%)**	9.59 (+ 5.0%)
two-layer	*9.00*	**9.33 (+11.2%)**	**9.43 (+10.5%)**	**9.48 (+8.9%)**	**9.60 (+8.18%)**	**9.70 (+7.22%)**

Table 5.1: Average Mean Average Precision (MAP) for predicting *co-purchase links* between Amazon products, when the size of co-purchase training set varies from 30% to 80% of the entire network. The joint model uses both co-purchase and category links (training set for the latter is always 80% of the network). Bold results are significantly better (t-test, $p < 0.001$).

Model	**Proportion of training set**					
	0 (transfer)	**0.3**	**0.4**	**0.5**	**0.6**	**0.8**
Cosine Similarity	7.50	7.50	7.50	7.50	7.50	7.50
SVMrank	–	7.73	7.82	7.89	7.92	7.92
Separate Model	–	7.86	7.99	8.07	8.38	8.61
Shared Similarity	*7.97*	**8.31 (+5.7%)**	**8.42 (+5.3%)**	**8.53 (+5.7%)**	**8.73 (+4.1%)**	**8.92 (+ 3.6%)**
two-layer	*8.48*	**8.79 (+11.8%)**	**8.8(+10.51%)**	**8.8(+9.66%)**	**9.12(+8.8%)**	**9.25(+7.4%)**

Table 5.2: Average Mean Average Precision (MAP) for predicting *category links* between Amazon products, when the size of category training set varies from 30% to 80% of the entire network. The joint model uses both co-purchase and category links (training set for the former is always 80% of the network). Bold results are significantly better (t-test, $p < 0.001$).

The ‘separate’ model shown in Table 5.1 uses only the link type that must be predicted and is identical to the classical distance learning model that has been used in previous works Bai et al. [2010], Shaw et al. [2011], Bai et al. [2009]. This model was previously compared with state-of-the-art link prediction methods and was shown to be effective for link prediction on networks. The separate model is ignorant about the other link type. The joint models make use of both link types according to the models described in this chapter. Table 5.1 clearly shows that the joint models improve the results significantly comparing to the separate model, SvmRank and Cosine similarity. The separate model is itself superior to the other models tested in Bai et al. [2010], Shaw et al. [2011], Bai et al. [2009]). SVMrank Joachims [2002], Herbrich et al. [2000] has shown high performance in ranking; to make it applicable to large graphs (here, the Amazon products) the optimization was performed on the primal form by using stochastic gradient descent. Not surprisingly, the two-layer Similarity model has better performance in comparison to the Shared Similarity model.

Table 5.2 shows similar results, but reversing the link types. In this experiment, we varied the proportion of category links made available for training, from 30% to 80% of the total number of objects, and kept the co-purchase links at a constant value (all links over 80% of the nodes used for training).

The important observation is that when there are few links available for training (columns at the left of the tables), the improvement of using both link types by the joint ranker is higher, percentage of the improvement is given in the parenthesis. However, even when the training set is large, the joint modeling is useful. Moreover, we observe in the extreme case when there is no training data available (marked “0 (transfer)” in the tables) and the ranker is only trained on the other link type, still the performance reached through *transferring* the learning from the other link type is significantly higher than the ad-hoc similarity metric in baselines (cosine-similarity). In the two-layer Similarity model we transfer the first layer and use an orthogonal matrix with all elements one for the second layer. The results confirm that there is correlation between categorical similarity of the products in Amazon and being co-purchased by customers, and each of them can help in modeling and predicting of the other one.

Table 5.3 similarly shows the average MAP of the TED dataset for the content links and Table 5.4 shows the average MAP for the co-liked links. The Joint models, specially two-layer model, significantly improve the link prediction performance for both link types. This shows that there is a big correlation between relatedness of talks in TED platform and being in the favorite list of many users.

Model	**Proportion of training set**					
	0 (transfer)	**0.3**	**0.4**	**0.5**	**0.6**	**0.8**
Cosine Similarity	4.55	4.55	4.55	4.55	4.55	4.55
SVMrank	–	5.10	6.44	6.87	7.44	8.02
Separate Model	–	4.94	5.78	6.49	8.18	9.28
Shared Similarity	*6.30*	**5.96 (+20.6%)**	**6.98 (+20.7%)**	7.52 (+ 15.8%)	9.27 (+ 13.3%)	10.38 (+ 11.8%)
two-layer	*7.61*	**8.72(+76.5%)**	**8.95(+54.8%)**	**9.50(+46.3%)**	**9.91(+21.1%)**	11.13(+19.9%)

Table
both content and co-liked links, the size of co-liked links is 0.8. Bold results are significantly better (t-test, $p < 0.001$).

Model	**Proportion of training set**					
	0 (transfer)	**0.3**	**0.4**	**0.5**	**0.6**	**0.8**
Cosine Similarity	2.84	2.84	2.84	2.84	2.84	2.84
SVMrank	–	2.40	2.72	2.73	2.33	3.77
Separate Model	–	3.42	3.75	4.02	4.82	5.70
Shared Similarity	*4.77*	4.14 (+21.0%)	**4.54 (+21%)**	**4.83 (+20.1%)**	**6.10 (+26.5 %)**	6.44 (+ 12.9%)
two-layer	*5.50*	**6.11(+78.6%)**	**6.22(+65.8%)**	**6.49(+61.4%)**	**6.97(+44.6%)**	**7.60(+33.3%)**

Table 5.4: Average Mean Average Precision (MAP) of *co-liked* links in TED when the size of co-liked train set is changed. The joint model uses both content and co-liked links, the size of content links is 0.8. Bold results are significantly better (t-test, $p < 0.001$).

The joint models have a higher performance than the separate models on both networks that we experimented with. The difference in the performance of the joint model and separate model is higher when there are fewer number of links from one link type. In that case, the denser link type can help more strongly the sparser link type through the shared information part. But even when there are equal links from different link types, joint modeling through shared information helps with the generality of the model and improves the link prediction performance.

5.3.4 Effect of Low-rank Factorization Dimension

We finally study the effect of low-rank factorization on the performance of ranking when the full training set is available. When the graph is larger, it is possible to train larger dimensions, but when the network is not large, larger dimensions make over-fitting more likely.

Table 5.5 shows the effect of dimension on link prediction performance for Amazon products. We observe that the growth of increase in the performance decreases when we increase the dimension. The time complexity of the algorithm linearly depends with the dimension. Therefore, for large scale graphs, one should choose a dimension by considering the trade-off between the time complexity and the performance.

Table 5.6 similarly shows the effect of dimension on the link prediction performance over the TED talks. Considering the size of network, increasing the dimension decreases the performance, as it over-fits the training set.

	Dimension			
Model	**20**	**50**	**100**	**150**
Separate Model (co-purchase)	6.60	8.52 (+29.9%)	9.13 (+7.16%)	9.31(+1.97%)
Shared Similarity (co-purchase)	7.29	9.24 (+26.75%)	9.59 (+3.79%)	9.91(+3.34%)
two-layer (co-purchase)	7.29	8.94(+22.63%)	9.70 (+8.50%)	10.11(+4.22%)
Separate Model (category)	7.59	8.41 (+10.08%)	8.61 (+2.38%)	8.67(+0.70%)
Shared Similarity (category)	7.96	8.76 (+10.05%)	8.92 (+1.83%)	8.98(+0.67%)
two-layer (category)	7.96	8.84(+11.05%)	9.25 (+4.63%)	9.40(+1.62%)

Table 5.5: Average Mean Average Precision (MAP) for predicting links between Amazon products.

5.4 Convergence of the Training Shared Similarity Model

To prove the convergence, first we need to establish an upper bound for the norm of the subgradients at each iteration $|| \bigtriangledown_t (.)||_F^2 < U$ which can be easily found if the data points are bounded. For example, if all data points vectors are normalized, then they are bounded, which is the case in our study.

Theorem 2. *Let $\bar{G} = \frac{1}{T}\sum_{t=1}^{T} G^t$ and $\bar{S_i} = \frac{1}{T}\sum_{t=1}^{T} {S_i}^t$ be the averages of G and S_i so far. Let the*

	Dimension				
Model	**10**	**20**	**50**	**100**	**150**
Separate Model (content)	9.33	1**0.63**	9.52	9.47	9.28
Shared Similarity(content)	9.53	**11.39**	11.05	11.02	10.38
two-layer (content)	9.90	**11.72**	11.67	11.13	11.06
Separate Model (co-liked)	5.95	**6.48**	6.21	5.89	5.70
Shared Similarity (co-liked)	6.30	**6.96**	7.09	6.39	6.44
two-layer (co-liked)	7.63	**7.99**	7.97	7.60	7.67

Table 5.6: Average Mean Average Precision (MAP) for predicting links between TED videos.

update rate be $\eta_t = \frac{1}{\lambda t}$ *with* $\lambda \leq 1/4$. *Let also* $G^* = \arg\min_G L(G)$ *and* $S_i^* = \arg\min_{S_i} L(S_i)$. *If the data points are bounded, i.e.* $||x_a' x_b||_F^2 < R$ *with* $R \geq 1$*, then with a probability of at least* $1-\delta$*, we have (with* $r = 4R^2$*):*

$$L(\bar{G}) \leq L(G^*) + \frac{21 r \ln(T/\delta)}{\lambda T}$$

$$L(\bar{S_i}) \leq L(S_i^*) + \frac{21 r \ln(T/\delta)}{\lambda T}$$

Proof. Each of the $L_t(G)$ and $L_t(S_i)$ is $\lambda-$strongly convex as they consist of a part with a shape $\frac{\lambda}{2}||M||_F^2$ and a convex function (the average hinge function). According to the Theorem 1 in Shalev-Shwartz et al. [2011] the upper bound of the subgradients norms $|| \bigtriangledown L_t(S_i)||_F^2$, $|| \bigtriangledown L_t(G)||_F^2$ is $r = 4R^2$.

$L(S_i)$ and $L(G)$ can be considered as the loss function of a one-class PEGASOS algorithm for which the input data pints are $x_a'(x_c - x_b)$ for $(a,b,c) \in Train_i$. In Lemma 2 in Shalev-Shwartz et al. [2011] the inequalities stated in the theorem are proven for the PEGASOS loss function and are therefore valid for the $L(G)$ and $L(S_i)$. □

As a consequence of the above theorem we can write :

$$\bar{L} = L(\bar{G}) + \sum_i L(\bar{S_i}) \leq L(G^*) + L(S_i^*) + \frac{21(N+1) r \ln(T/\delta)}{\lambda T}$$

Therefore we can see that to obtain an error inferior or equal to ϵ with the confidence $1-\delta$ we need $\tilde{O}(\frac{N+1}{\epsilon\delta\lambda})$ iterations.

5.5 Conclusion

In this chapter, we proposed two joint similarity learning models over nodes' attributes for link prediction in networks with multiple link types. The first model learns a similarity metric that consists of two parts: the general part, which is shared between all link types, and the specific part, which is trained specifically for each type of link. The second model consists of two layers: the first layer, which is shared between all link types, embeds the objects of the network into a new space, and then a similarity is learned specifically for each link type in this new space. Both models are applicable to large networks with high-dimensional feature spaces. The experiments show that the proposed joint modeling and training frameworks improve link prediction performance significantly for each link type in comparison to multiple baselines. This improvement is higher when there are fewer links available from one link type in the network. The two-layer similarity model outperforms the first one which is expected due to its capability of modeling negative correlations among different link types. Moreover, we illustrated that even if the models are trained completely on one link type and tested on the other, our models significantly improve the performance in comparison to ad-hoc similarity metrics.

6 Similarity Learning for Collective Ranking on Networks: Application to Link Prediction

This chapter proposes a method for learning to rank over networks (relational data). The ranking is performed with respect to a query object which can be part of the network or outside it. The ranking method makes use of the features of the nodes as well as the existing links between them. First, a neighbors-aware ranker is trained using a large margin pairwise loss function. Then, collective inference is performed using an iterative ranking algorithm, which propagates the results of rankers over the network. By formulating link prediction as a ranking problem, the method is tested on three networks, with papers/citations and webpages/hyperlinks. The results show that the proposed algorithm, which uses both link structure and node attributes, outperforms several other methods: a content-only ranker, a link-only one, a random walk method, a relational topic model, and a method based on the weighted number of common neighbors. In addition, the propagation algorithm improves results even when the query object is not part of the network, and scales efficiently to large networks.

The chapter is organized as follows. After a brief introduction to the problem (Section 6.1), we frame the model and put it into perspective in Section 6.2. In Section 6.3 we explain the collective ranking framework, first by introducing the neighbors-aware ranker and then the collective inference algorithm. In Section 6.4 we evaluate the proposed method on the three different data sets, showing that it outperforms several other methods.

6.1 Introduction to Ranking on Relational Data

In network or relational data, the relations between objects are represented by directed or undirected edges in a network. The ranking problem on network data requires that all objects in the network are ranked given a query.

Two cases of the ranking problem can occur in network data such as social networks: the query object itself is an object in the network – i.e. some prior relations with other objects are known – or the query object is not part of the network.

When the query object has already some links in the network, link-based approaches such as random walk models can be effective, whereas when the query object is not in the network, these link-based approaches are not applicable, and usually supervised learning over node attributes are considered.

In both cases, it is possible to exploit the relations between objects in the network to increase the accuracy of the ranking with respect to a given query. In this work we bridge these two approaches and make use of attributes and of relations between objects at the same time.

For illustration purposes, let us consider a network formed by scientific papers linked by citations between them. Given a "query" paper, the goal is to rank all papers in the network according to a score that reflects the likelihood of being cited by this query paper. If the network is made of completed papers with optimal citations, then this score should distinguish the actually cited (respectively not cited) papers. Moreover, the model will also indicate papers that *should* have been cited and were not, or, conversely, citations that were *spuriously* inserted. But the ranking task is more clearly useful for recommending citations to include in a new paper (object outside the network) or to extend an existing draft (already in the network). In both cases, the model computes a ranked list of recommendations for additional citations, using existing links and object features as well. A similar application is the recommendation of new connections in social networks.

6.2 Motivation of the Model

Learning to rank can be formulated as a supervised learning task. Suppose that each object, including the query, can be represented as feature vectors in a given space. Let X be the list of feature vectors X_i for a list of objects, and let Y be the list of corresponding grades or relevance scores, for each object, with respect to a given query object q. For example, in the case of a friendship social network, the grades Y_i represent the friendship status ('friend' or 'not friend') with the query object (an individual profile) for each member i, and the feature vectors X_i are derived from the information available from individual profiles of each member i. In information retrieval terms, the grades Y represent the relevance degree of the objects with respect to the query.

The goal of the *learning to rank* task is to automatically learn a function F transforming a list of feature vectors X into a list of scores with respect to a query q, given the training data $(X_{q_i}, X_1, Y_{1q_i}), (X_{q_i}, X_2, Y_{2q_i}), \ldots, (X_{q_i}, X_m, Y_{mq_i})$ in which X_{q_i} and X_j are showing the feature vectors of the query q_i and the object j, and Y_{jq_i} shows their corresponding grade.

In all generality, $F(q, X)$ is a *global* ranking function, meaning that F assigns scores to the list of features vectors X for a given set of objects and a query. A global ranking function is able to consider dependencies between objects as it scores a set of objects instead of each object alone. However, to make the computation tractable in practice, a *local* ranking function is usually considered for applications:

$$F(q,X) = [f(q,X_1), f(q,X_2), \ldots, f(q,X_n)]$$

The implicit assumption behind using a local ranking function is that the score of each object is independent from the score of the other objects. In many applications this assumption yields acceptable results, but in network data this assumption is quite questionable. In a friendship network, for instance, friendship is more likely if there are more common friends, and not only if two people have similar profiles. Or, in a citation network, a paper is more likely to cite another paper if it has already cited several papers that cite the considered paper.

In this chapter, we model the dependency between data objects in the network to solve the ranking problem more effectively. We make use of a loss function $L(\cdot,\cdot)$ to evaluate the prediction result of $F(q,X)$ upon training. First, the feature vectors X are ranked according to their scores $F(q,X)$. Then, the ranking results are evaluated against the corresponding expected grades Y. If the feature vectors with higher grades are ranked higher, then the loss L will be small, otherwise it will be large.

However, the minimization of the loss function is difficult as it is not continuous and uses sorting Li [2011]. Therefore, we consider a surrogate loss function L' to make the minimization possible, specifically a pairwise hinge loss function. This function attempts to preserve the pairwise order between the objects in the training set by maintaining a margin between the scores of each pair Bai et al. [2010]. For instance, in a friendship social network, the hinge function aims to preserve the order between the scores of the friend pairs and the non-friend pairs.

Link prediction can also be formulated as a learning-to-rank problem: given a query node, all other nodes are sorted according to the likelihood of creating a link between them and the query node. Link prediction is a binary graded ranking and, in the training phase, if the linked objects are ranked higher, then the loss will be small, otherwise it will be large. This formulation has been used in many previous works including Liben-Nowell and Kleinberg [2003], Adamic and Adar [2001], Backstrom and Leskovec [2011], Shaw et al. [2011], Agarwal et al. [2006]. In the rest of this chapter we focus on the binary ranking problem for link prediction, although the framework can be easily generalized for other ranking problems.

6.3 Learning to Rank on a Network

The classification of objects from network data sets has been shown to be successfully addressed by using collective classification algorithms Jensen et al. [2004], in which a local classifier is trained to classify each object independently by using at the same time object features and labels of other objects in its neighborhood. A collective inference algorithm propagates the results of the local classifiers in the network.

For ranking on a network, we draw inspiration from this approach to define a collective ranking algorithm which consists of three parts:

1. The *content-only ranker* learns to rank only by using a similarity function on node attributes, while ignoring relations between nodes. This ranker embodies the approach which is traditionally used in learning to rank problems.

2. The *neighbors-aware ranker* learns how to rank according to the information locally available at each node, coming from the node's attributes and the neighborhood information. The size of the neighborhood of which the ranker is aware can vary in principle; however, in this work, we limit the neighborhood to the direct neighbors, i.e. one transition away.

3. The *collective inference algorithm* propagates the results of local rankers over the network. Existing collective inference methods McDowell and K.M. Gupta [2009] are used as inspiration for an original iterative propagation algorithm, defined below. We will show that this algorithm is effective for collective ranking and is simpler to implement and scales better to larger graphs in comparison to other collective inference algorithms. Moreover, we prove the convergence of the inference algorithm.

In the following subsections we describe the neighbors-aware local ranker, the collective inference algorithm, and the training process for the ranker.

6.3.1 Neighbors-aware Ranker

The neighbors-aware ranker returns a score given a query, a target node, and the overall graph structure. The ranker's score is a function of the neighbors' scores and of the similarity between the query and the target node. If we denote the ranker by f, the entire graph by G, the query and target nodes respectively by q and t, and the features of a node i by x_i, then we have:

$$f(q,t,G) = \underbrace{similarity(x_q,x_t)}_{\text{content-only}} + \alpha \underbrace{neighbor(q,t,G)}_{\text{neighbors scores}}$$

The first part, $similarity(x_q,x_t)$, is the content-only ranker which computes the similarity between q and t based only on their attributes, as proposed by most previous learning to rank methods. Many content-based rankers with various learning abilities have been studied in the literature, for instance RankNet Burges et al. [2005], polynomial semantic indexing Bai et al. [2009] or structure preserving metric learning Shaw et al. [2011]. In this study, we choose a similar approach and define the similarity function as:

$$similarity(x_q, x_t) = x_q \times M \times x_t'$$

The M matrix is a $N \times N$ matrix, where N is the number of words. In practice, to make the training and the storage possible, some constraints on M are considered, such as using a diagonal matrix or performing low-rank factorization Bai et al. [2009], Shaw et al. [2011].

The second part of f, i.e. the neighborhood score, shows how the neighbors of t are scored with respect to q. If the neighbors of t have high scores, it is more likely that t itself has high scores. For instance, in a friendship social network, a person q is more likely to be friend with a person t if they have many common friends. This part of the neighbors-aware ranker takes into account the dependencies between data objects, and allows us to perform collective ranking instead of local ranking only. As we are dealing with directed networks here, we consider two types of neighbors: neighbors from in-links ($\mathcal{N}_{in}(t)$) and neighbors from out-links ($\mathcal{N}_{out}(t)$). We define the $neighbor$ function recursively by using the score function of the neighbors as follows:

$$neighbor(q, t, G) = w_1 \frac{\sum_{n \in \mathcal{N}_{in}(t)} f(q, n, G)}{|\mathcal{N}_{in}(t)|} + w_2 \frac{\sum_{n \in \mathcal{N}_{out}(t)} f(q, n, G)}{|\mathcal{N}_{out}(t)|}$$

The parameters w_1 and w_2 represent the importance of the in-links neighbors and out-links neighbors, and are learned during the training along with the parameters of the $similarity$ function.

Putting all together we have the following formula for the neighbors-aware ranker:

$$f(q, t, G) = x_q M x_t' + \frac{\alpha w_1 \sum_{n \in \mathcal{N}_{in}(t)} f(q, n, G)}{|\mathcal{N}_{in}(t)|} + \frac{\alpha w_2 \sum_{n \in \mathcal{N}_{out}(t)} f(q, n, G)}{|\mathcal{N}_{out}(t)|}$$

According to the above formula, the score of a target node t is defined based on the score of its neighbors, recursively. Therefore, to compute the score to a target node, the score to its neighbors should be computed, and to compute the score to the neighbors, the score to the neighbors of the neighbors should be computed first, and so on. The next sections explains how these values are estimated.

The α parameter in the definition of the neighbors-aware ranker (we choose $\alpha < 1$) is the dampening parameter which decreases the effect of the neighbors' scores on the target node score as their distance from the target node increases. For instance, the effect of a neighbor two transitions away is α times smaller than the effect of the neighbor one transition away.

A larger α means that a larger neighborhood is allowed to have an effect on the target node score, whereas a smaller α achieves the opposite effect. Therefore, α is a hyper-parameter of the algorithm and is not learned during the training.

6.3.2 Collective Inference

To compute the score for a given query and a target node, we need to know the scores of the query to the target's neighbors. To compute this recursive function, we start from an initial score and iteratively compute the scores.

The score of a node at iteration τ is thus computed based on the score of nodes at iteration $\tau - 1$. More precisely, if $f^{\tau}(q,t,G)$ is the score of node t for the query q at iteration τ, it is computed as:

$$f^{\tau}(q,t,G) = x_q M x_t' + \frac{\alpha w_1 \sum_{n \in \mathcal{N}_{in}(t)} f^{\tau-1}(q,n,G)}{|\mathcal{N}_{in}(t)|} + \frac{\alpha w_2 \sum_{n \in \mathcal{N}_{out}(t)} f^{\tau-1}(q,n,G)}{|\mathcal{N}_{out}(t)|}$$

There are two cases for the initial scores $f^0(q,t,G)$: first, if no prior relation is known for the query in the network, the scores are initialized by the scores of the content-only ranker. Second, when the query node itself is in the network, they come from the prior known relations, the initial score is set to 1 if (i,j) is an edge of the graph G and to 0 otherwise.

To perform the propagation, we propose here a collective inference algorithm, which is effective for collective ranking, as we will show. The algorithm is easy to implement and scales well to large graphs in comparison to other collective inference algorithms. We refer to the algorithm as 'Iterative Ranking Algorithm' (IRA) because at each iteration the results of the neighborhood-aware rankers are propagated one step further in the graph. The pseudocode for the IRA is given as Algorithm 7 below.

At the first step, the initial score of all nodes with respect to the query is computed according to the explanation above. Then, scores are normalized and the scores above the threshold propagate to the neighbors at the next step of the algorithm. This propagation of above-threshold scores continues until convergence or until the maximum number of iterations is reached.

The algorithm converges if $|w_1| < 1$ and $|w_2| < 1$, because the effect of long paths decreases exponentially and eventually vanishes. To ensure the convergence, during the training of the ranker we impose the constraint that all the parameters (similarity matrix parameters and w_i) must be between zero and one. By imposing this constraint on the parameters we achieve two goals: (1) we can prove the convergence, and (2) the constraint acts as a regularizer on the parameters in the loss function minimization and avoids arbitrary growth of the parameters (see Section 6.3.3).

Algorithm 7 Iterative Ranking Algorithm.

```
IRA for query node $q$
$q$ = query node,
$f$ = neighborhood-aware ranker,
$f_{content}$ = content-only local ranker,
$c$ = threshold for scores to propagate,
$T$ = maximum number of iterations,
$G$ = graph with vertex set $V$ and edge set $E$,
$\alpha$ = dampening parameter,
$f_i^\tau$ = score of node $i$ after $\tau$ iterations (long: $f^\tau(q,i,G)$)
**if** $q \in V$ **then**
  $\forall i \in V, f_i^0 = \begin{cases} 1 & \text{if } (q,i) \in E \\ 0 & \text{otherwise} \end{cases}$
**else**
  $\forall i \in V, f_i^0 = f_{content}(q,i)$
**end if**
$f_{norm}^0(i) = norm(f_i^0)$ (normalize scores to [0, 1])
$\tau = 1$
**while** NotConverged and $\tau \le T$ **do**
  **for** $i \in V$ **do**
    **if** $f_{norm}^{\tau-1}(i) < c$ **then**
      $f_i^{\tau-1} = 0$
      (scores below threshold do not propagate)
    **end if**
  **end for**
  $HighScoreNodes = \{i | f_i^{\tau-1} > 0\}$
  $PossiblyChanged = \{i | i \in \mathcal{N}_{in}(HighScoreNodes) \vee i \in \mathcal{N}_{out}(HighScoreNodes)\}$
  **for** $i \in PossiblyChanged$ **do**
    $f_i^\tau = f(q,i,G,f_i^{\tau-1})$
  **end for**
  $f_{norm}^\tau(i) = norm(f_i^\tau)$ (normalize scores to [0, 1])
  $\tau = \tau + 1$
**end while**
```

The proof of convergence of the algorithm can be sketched as follows. We first make explicit the value of f_i^τ, the score of the node i after τ iterations, and we omit q from the equations for simplicity.

$$\begin{aligned} f_i^\tau &= Sim_i + \alpha \frac{w_1}{|\mathcal{N}_{in}(t)|} \sum_{n_1 \in \mathcal{N}_{in}(t)} \left(Sim_{n_1} + \alpha \frac{w_1 \sum_{n_2 \in \mathcal{N}_{in}(n_1)} f_{n_2}^{\tau-2}}{|\mathcal{N}_{in}(n_1)|} + \alpha \frac{w_2 \sum_{n_2 \in \mathcal{N}_{out}(n_1)} f_{n_2}^{\tau-2}}{|\mathcal{N}_{out}(n_1)|}\right) \\ &+ \alpha \frac{w_2}{|\mathcal{N}_{out}(t)|} \sum_{n_1 \in \mathcal{N}_{out}(t)} \left(Sim_{n_1} + \alpha \frac{w_1}{|\mathcal{N}_{in}(n_1)|} \sum_{n_2 \in \mathcal{N}_{in}(n_1)} f_{n_2}^{\tau-2} + \alpha \frac{w_2}{|\mathcal{N}_{out}(n_1)|} \sum_{n_2 \in \mathcal{N}_{out}(n_1)} f_{n_2}^{\tau-2}\right) \end{aligned}$$

If we continue expanding the above equation, the coefficient for the similarity of a node that is n transitions away is $\alpha^n \times (w_{a_1} \cdots w_{a_n})$ with a_i being either 1 or 2. When increasing n, this coefficient tends to zero (given that $\alpha < 1$, $w_1 < 1$, $w_2 < 1$, etc.), which means that the effect of long paths (including loops) vanishes, and the algorithm converges. One might think that the thresholding changes this linear algebraic nature of the propagation and then the convergence can not be guaranteed, but the normalization of the scores and thresholding are used in the following way considering this issue. In algorithm 7, if the normalized score is higher than the threshold, the *unnormalized* score is passed to the next step, and otherwise the score is set to zero in the next step. In other words, in the above expansion only some terms are expanded and others are set to zero, which still holds the convergence.

The scores below threshold c will not propagate to the next step of the IRA. There are two main reasons to add this threshold: the first one is that it prevents the propagation of noise in the graph, therefore improving performance, as shown in Section 6.4.1. The second reason is to increase the speed of the collective inference algorithm for larger graphs. In the IRA, in each iteration, only the score of nodes for which the scores are possibly changed are updated. Only the scores of the neighbors of nodes with scores higher than the threshold can be changed in the next iteration. By using a reasonably high threshold, only the scores of very few nodes will change, despite the size of the graph, and therefore the propagation algorithm is sped up, as confirmed experimentally in Section 6.4.1. However, in theory, the worst case time complexity of the algorithm does not change and is still $O(NT)$.

The above reasons to use threshold in the propagation algorithm are justified, if and only if, there are only few nodes with high scores in each iteration and there is a long-tail of low scores nodes. Most real-world networks are *small world* networks, meaning that nodes form *communities*, and these communities are connected by *short paths*. Therefore, each node is not connected uniformly to all other nodes in the network, but is usually only connected to a small number of other nodes in its community. We perform more experimental investigations in the next sections to validate these assumptions on the networks used as test sets.

6.3.3 Training the Neighbors-aware Ranker

The goal of training is to approximate a local ranker which minimizes a ranking loss function over the graph G for the queries, target nodes, and associated grades that are given in the training set. In this section we discuss the training of the binary ranking for link prediction problem. This can be easily generalized to other ranking problems if needed.

At training time, the graph $G = (V, E)$ is available, where V is the vertex set and E the edge set, as in Algorithm 7. We assume that there are two possible grades, 'connected' or 'not connected', which are given by the existing edges in the graph. We consider the following training set $\mathscr{T}$ with triples of nodes such as $\mathscr{T} = \{(i, j, z) | (i, j) \in E \,\text{and}\, (i, z) \notin E\}$. The goal of training is to minimize the empirical risk using a pairwise hinge loss function L over the training set $\mathscr{T}$ as follows:

$$L = \sum_{(i,j,z)\in\mathcal{T}} max(0, d_{marg} - f(i,j,G) + f(i,z,G))$$
$$\text{so that } 0 \leq w_i \leq 1 \text{ and } 0 \leq M_{ij} \leq 1$$

where w_i are the parameters of the neighborhood component of f and M_{ij} are the parameters of the similarity component of f. We define d_{marg} as a constant margin that should separate the examples of two grades (connected vs. not connected). By using the constraint that the parameters should be bounded, we ensure that the collective inference algorithm converges. Moreover, the constraint is a regularization on the parameters and prevents the arbitrary growth of the parameters in the optimization, which is equivalent to the optimization of the same loss function plus the infinity norm of the parameters.

If the training graph G is large, then minimizing the above summation is not tractable. To overcome this problem we make use of a stochastic gradient descent algorithm in which at each iteration we randomly select i, j and z from G and perform gradient descent on them. To impose the constraint we use gradient projection method.

To perform the training for a given triple (i,j,z), we need to know the score of query i to the neighbors of j and z. We set the score to the maximum value (1) if there is a link between i and the neighbor of j in the training graph G, and to the minimum value (0) if there is no such link. In this case, the optimization is convex and the gradient of the loss function is easy to compute as the score of the neighbors is constant. We use $E_{ij} = 1$ if $(i,j) \in E$ and $E_{ij} = 0$ otherwise. The subgradients of the approximate objective function at time (iteration) τ are the following:

$$(i,j,z) \in \mathcal{T}$$
$$\triangledown L_\tau(M) = \{ \begin{array}{ll} x_i'(x_j - x_z) & \text{if } f^\tau(i,j,G) - f^\tau(i,z,G) < d \\ 0 & \text{otherwise} \end{array}$$
$$dif_{in} = \frac{\sum_{n\in\mathcal{N}_{in}(j)} E_{in}}{|\mathcal{N}_{in}(j)|} - \frac{\sum_{n\in\mathcal{N}_{in}(z)} E_{in}}{|\mathcal{N}_{in}(z)|}$$
$$\triangledown L_\tau(w_1) = \{ \begin{array}{ll} dif_{in} & \text{if } f^\tau(i,j,G) - f^\tau(i,z,G) < d \\ 0 & \text{otherwise} \end{array}$$
$$dif_{out} = \frac{\sum_{n\in\mathcal{N}_{out}(j)} E_{in}}{|\mathcal{N}_{out}(j)|} - \frac{\sum_{n\in\mathcal{N}_{out}(z)} E_{in}}{|\mathcal{N}_{out}(z)|}$$
$$\triangledown L_\tau(w_2) = \{ \begin{array}{ll} dif_{out} & \text{if } f^\tau(i,j,G) - f^\tau(i,z,G) < d \\ 0 & \text{otherwise} \end{array}$$

The update of the parameters is done consequently as follows:

$$M^{\tau+1} = M^{\tau} - \eta_\tau \bigtriangledown L_\tau(M)$$
$$w_1{}^{\tau+1} = w_1{}^{\tau} - \eta_\tau \bigtriangledown L_\tau(w_1)$$
$$w_2{}^{\tau+1} = w_2{}^{\tau} - \eta_\tau \bigtriangledown L_\tau(w_2)$$

where η_τ is the learning rate at the iteration τ.

The dimension of M is N^2 where N is the number of features of the data points. In practice, to make training and storage possible, particularly when we are dealing with high dimension data such as text, we perform low-rank factorization of M. In this case we assume that $A = AB'$ so that A and B are matrices from N to a lower dimension. Given that M is not necessarily symmetric, we decompose it into two lower-rank matrices. The objective function consisting of the low-rank matrices is not convex any more, but in practice it has been shown that the low-rank factorization performs well Bai et al. [2009]. At each iteration the time complexity of the stochastic training algorithm is $O(Z^2)$, where Z is the average number of non-zero features of the objects, which can be much smaller than the dimension N of the features. For example, for textual data, the feature vectors are very sparse in comparison to the dimension of the features (i.e. the number of possible words, or vocabulary size).

6.4 Experimental Setup and Results

In this section, we apply the proposed method to Link Prediction problem (connected vs. not connected) on three network data sets: WEBKB, CiteSeer and Cora which have already been explained in Section 3.3.3. Given a query, all other objects are ranked according to their score, and the goal is to get the linked objects at the top of this list.

To build test sets, we randomly exclude some objects from the initial network (about 10%), and train the algorithm on the remaining network.

We present the performance of the proposed method in terms of *precision* and *recall at 10* for the three data sets. The training set is built by excluding about 10% of the nodes and their links from the network, meaning that for each query object there is no prior link in the training network. This scenario corresponds to certain real-world situations, for example when a new person joins a social network, or a paper is being written and does not cite any other paper yet. Therefore, the ranking is performed by using two types of information sources only: first, the features of the query node and of the objects in the network; and second, the relations between the objects in the network. The second type can be exploited only by using the neighboorhood-aware ranker and the propagation algorithm described above. This task is clearly difficult for algorithms based on only the link structure, for example random walk algorithms or algorithms based on common neighborhood.

Another possible real-world situation is when some prior relations of the query object are known, e.g. a paper with some known citations or a person which already has some relations in a social network. This will also be explored, by making available some of the relations of the query object.

To perform a detailed analysis of the performance of the proposed algorithms, their performance is analyzed by changing the proportion of the known prior links of the query objects in the test set, from 0 to 0.7, with 0 meaning that no prior links of the query object are known (first scenario). This covers therefore the two types of situations described above.

The threshold of the IRA algorithm is set to $c = 0.8$, which makes the computation fast on the studied networks, and the effect of this threshold is discussed in the following section through additional experiments. The similarity matrix M is constrained to be diagonal. The reported results are the *average of 10 different runs* for which the test set was chosen randomly at the start of each run.

Figures 6.1 and 6.2 show respectively the average precision and recall at 10 for the eight algorithms listed below, where the first one corresponds to the full proposal of this chapter, the second one to its first part only (without propagation), and the other six are previously proposed ones, serving for comparison. Namely, we report the performance of the Personalized Page Rank random walk Haveliwala [2003], of the Adamic and Adar similarity Liben-Nowell and Kleinberg [2003], and of a relational topic model, which are the methods that gave especially high scores in previous studies.

1. Neighbors-aware ranker with IRA.
2. Neighbors-aware ranker without IRA.
3. Content-only ranker.
4. Neighbors-only ranker with IRA.
5. Neighbors-only ranker without IRA.
6. Relational Topic Model (RTM).
7. Personalized Page Rank random walk (RW).
8. Adamic and Adar similarity.

Figures 6.1 and 6.2 show that for all three data sets the ranking performance using the neighbors-aware ranker with the propagation algorithm ,IRA, is always higher than all the other algorithms, including the neighbors-aware ranker alone. Therefore, the method proposed in this chapter appears to be effective and competitive.

When the number of known relations of the query objects is increased (right side of the graphs), the performance of the neighbors-only ranker and the link-based measures get closer to the rest of the rankers (such as random walk), because the neighborhood and link structure information become available. But when there are few known relations of the query objects in the network (left side of the graphs), the performance of the neighbors-aware ranker plus IRA, neighbors-aware ranker alone and even content-only ranker are much higher than link-based approaches such as random walk model.

Propagation by IRA is shown to be useful even when queries have no known relations at all in the network (leftmost data points in Figures 6.1 and 6.2). In this case, the IRA propagates the result of the content-only ranker in the network by exploiting the relations between objects in the network. For instance, this can increase the score of a node that does not get a high score from features similarity, but is connected to many nodes with high scores. If we imagine the scientific papers network again, a low-score paper (according to the content) might get a high score after propagation because it is cited by many high-score papers. Therefore, modeling the relations between objects in the network even when there are no known links for the query object is shown to be helpful.

Neighbors-aware ranker, that uses neighborhood information that is only one transition away, improves the results significantly for Cora and CiteSeer networks, but does not always improve the results on WEBKB network. On the other hand, the propagation algorithm, IRA, improves the performance more on WebKB and Cora network and is less effective on CiteSeer network. The combination of both neighbors-aware ranker and IRA is a rich model that is able to model varying-length dependencies if needed, and results in a robust ranking method which is effective on different networks.

Precision and recall score are not necessarily increasing with the increase of the known links of the query objects (going from the left to the right side of the figures). This is because for each query object in the dataset only a fixed number of connected objects (e.g. cited papers) is available, and by revealing more of them the number of potential correct answers decreases, i.e. the upper bound of precision at k.

6.4.1 Effect of Threshold

In this section, we analyze experimentally the effect of the threshold c for the propagation of scores in the IRA algorithm. We report *precision and recall at 10* for the three test data sets while varying the threshold c and the proportion of known links, over the three data sets, in Figures 6.3 and 6.4.

The first observation is that the performance increases when increasing the threshold. The main reason is that with a lower threshold, many wrong predictions are propagated in the graph. Still, when the threshold is close to 1, the performance drops again as the propagation (which was shown above to be clearly useful) decreases and eventually stops.

The results demonstrate the advantage of the proposed method: the performance increases when using a high threshold, and in this case only few nodes are updated at each iteration, therefore the IRA is much faster. The running time, in practice, is nearly constant with respect

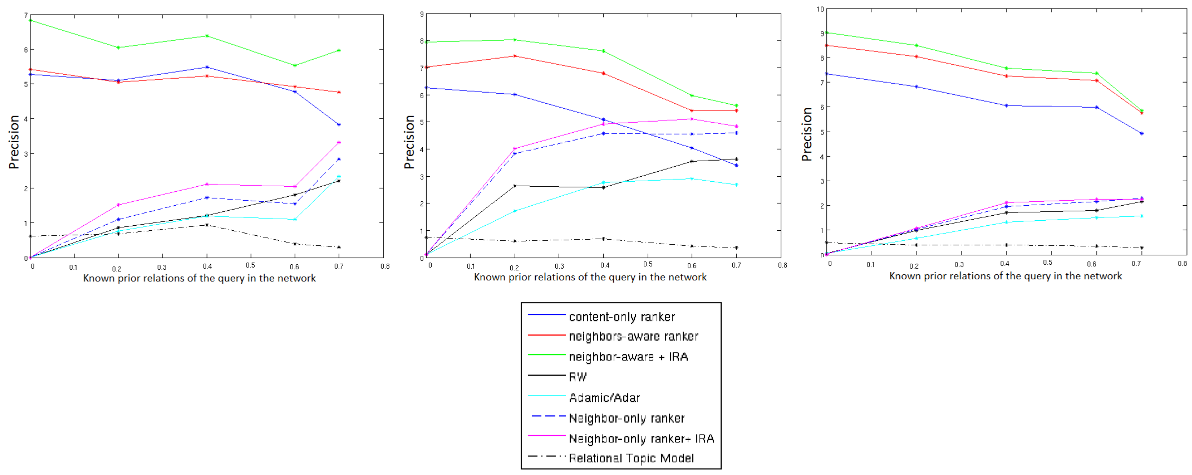


Figure 6.1: Precision at 10, as a percentage, for the eight rankers listed in the text. The proportion of known prior relations of the query objects in the network is increased from 0 to 0.7. From left to right, the data sets are WebKB, Cora, and CiteSeer.

to the graph size, although the theoretical worst case time complexity remains linear. We report in Figure 6.5 the required inference time on the data sets while varying the threshold c. Increasing the threshold makes the required time decrease rapidly for low thresholds, as there are many nodes with very low scores (long tail). Then, by increasing further the threshold, the required time decreases only slightly. This confirms our assumption that the networks are *small world* graphs and form connected communities, which makes our propagation algorithm very efficient.

We performed a similar experiment varying α when the threshold c is fixed. The results show that for very small values of α the neighborhood does not have any effect on the score and performance unsurprisingly decreases. If α is close to 1, then although the w_1 and w_2 are trained on the network, the training becomes more difficult as the loss function gives a high weight to the neighborhood part in comparison to the similarity part, so again performance decreases. In addition, when $alpha$ is close to 1, the convergence is harder and loops can have negative effects on the performance.

6.5 Conclusion and Future Perspectives

We proposed a learning to rank algorithm on network data, which uses both the attributes of the nodes and the structure of the links, for learning and inference. The proposed collective inference algorithm, IRA, propagates the predicted scores through the graph on condition that they are above a given threshold. Thresholding adds two features to the algorithm: it improves performance, and makes a time-efficient implementation possible, for application to large scale graphs.

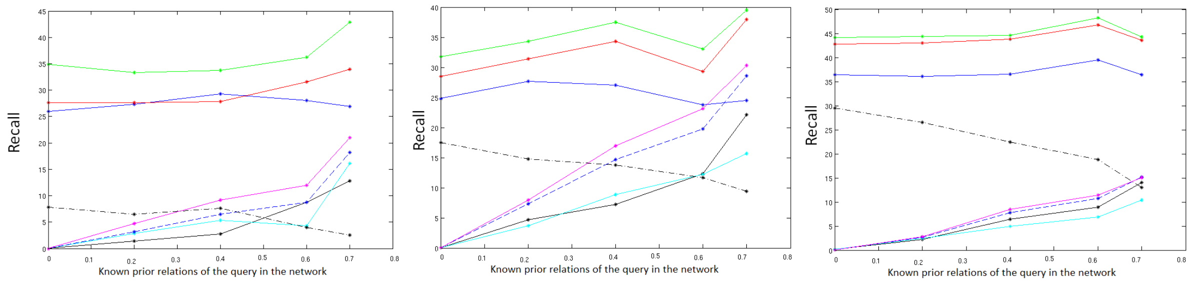


Figure 6.2: Recall at 10, as a percentage, for the eight rankers listed in the text. The proportion of known prior relations of the query objects in the network is increased from 0 to 0.7. From left to right, the data sets are WebKB, Cora, and CiteSeer.

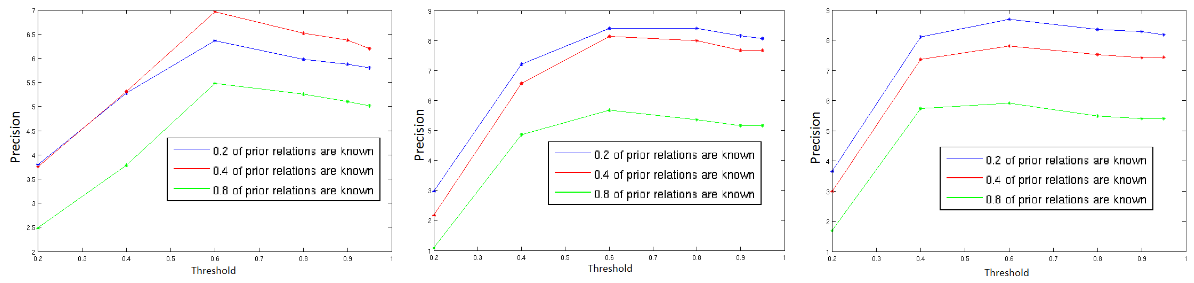


Figure 6.3: Precision at 10, as a percentage, when increasing the threshold c of the IRA from 0.2 to 0.95 on the three data sets. In each graph, the curves correspond to 0.2, 0.4 and 0.8 of prior relations known.

The experimental results showed that using the proposed algorithm improves the performance of the link prediction on three different network data sets, for binary graded scores (connected vs. not connected). The results showed more specifically that the neighbors-aware ranker, which uses content features and scores of the neighbors, has a higher performance than the content-only ranker, the neighbors-only ranker, Relational Topic Model and two linked-based similarity measures. Moreover, using the proposed propagation algorithm, IRA, in addition to the neighbors-aware ranker improves the performance even more.

In the current model, the influence of the neighbors is not considered: all neighbors are assumed to be equal. However, neighbors might have different influences, including negative ones. Our model can be generalized to learn a node-specific influence of neighbors as a function of their features and of the features of the edges between them. This model has more parameters and needs therefore bigger training sets to perform reliable experiments.

Moreover, a recursive procedure can be considered for training the ranker. Here, when training over a network, we assumed that the scores of a target node's neighbors are constant (1 if they are connected with the query or 0 otherwise). The implicit assumption is that the

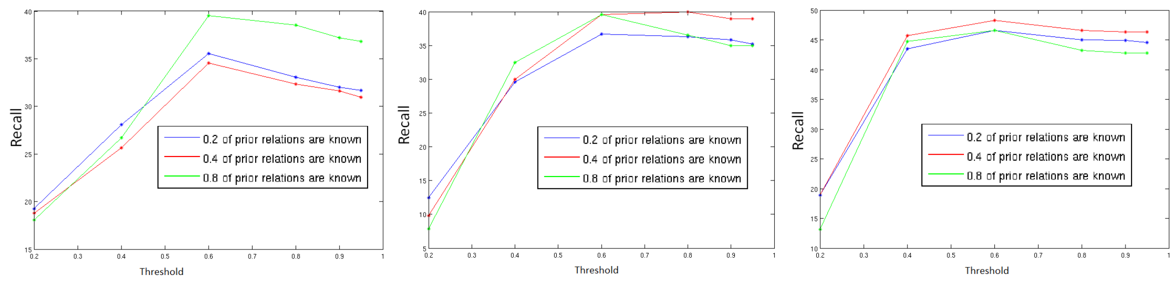


Figure 6.4: Recall at 10, as a percentage, when increasing the threshold c of the IRA from 0.2 to 0.95 on the three data sets. In each graph, the curves correspond to 0.2, 0.4 and 0.8 of prior relations known.

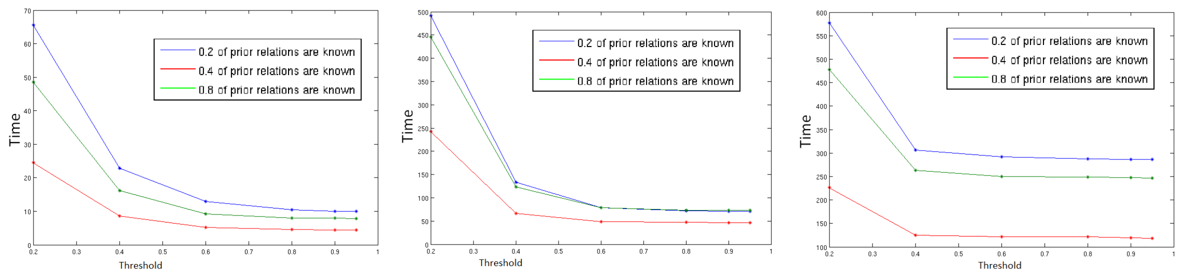


Figure 6.5: Inference time of the IRA when increasing the threshold c from 0.2 to 0.95 on the three data sets. In each graph, the curves correspond to 0.2, 0.4 and 0.8 of prior relations known.

training network is “complete” and all links are “correct”. In some networks this assumption is questionable, since there are missing links or spurious ones. To model this fact, it is possible to train the ranker recursively: first by using the constant scores according to the current links in the network, and then by replacing these scores by the scores provided by the ranker learned in the previous iteration. This model also deserves further experimental investigations.

7 Conclusion and Perspectives

In this thesis, we have proposed novel solutions to similarity learning problems on collaborative networks. As shown throughout the thesis, similarity learning is an essential task for modeling and predicting the evolution of collaborative networks. Moreover, we showed that similarity learning is used to perform ranking, which is the main component of recommender systems. Due to the the low cost of developing such collaborative networks, they grow very quickly, and therefore, we have aimed for models that scale well to large networks.

Conclusion

Various sources of information can be considered when building a distance measure over a collaborative network. The first important source of information is the *global* link structure of the network. In collaborative networks, there are many spurious and missing links, which make a proximity measure based on local link structure unreliable. The link structure also might consist of multiple link types.

In Chapter 3, to answer these requirements, we defined a random walk model, named Visiting Probability (*VP*), to measure proximity between two nodes in a graph. *VP* considers all the paths between two nodes collectively and thus reduces the effect of unreliable links. We defined a symmetric *VP* similarity measure and showed experimentally that using the symmetric *VP* improves the prediction performance of the distance. Moreover, we showed how to use both the *VP* definition and the structural characteristics of many social networks, namely the case of small-world networks, to design scalable algorithms based on *VP* similarity. We designed approximation algorithms to perform ranking based on *VP* on large graphs. Besides, we defined the community of a node according to *VP* and gave experimental evidence that this definition was effective. Fast algorithms were designed to solve K-nearest neighbors and identify the communities over large graphs, based on symmetric *VP* proximity.

In the second part of Chapter 3, we took advantage of the homophily principle in social networks, the fact that two nodes having similar ”social characteristics” have a higher probability to be linked. We defined a latent space as the space that is formed by the “social characteristics” or “latent features”, and assumed that homophily holds in this latent space.

Therefore, we showed that the link structure of a graph can be modeled by the proposed similarity learning framework, in which the transformation of nodes to the latent space is trained using a discriminative model. We showed how to apply this framework to learn the similarity between two nodes based on their attributes, over large graphs. Moreover, we showed that similarity learning on attributes derived from random walk scores, specifically *VP* scores, can model and predict better the relations in the graph in comparison to learning on the network’s links directly. Especially, we showed that training the latent space on the *VP*-based communities of nodes resulted in the best prediction performance.

Therefore, we used both global network structure (*VP* scores) and node attributes to learn a reliable similarity measure. In the experimental results, we observed that sometimes the nodes’ attributes were not predictive enough to be able to describe the global link structure of the network, and the prediction performance dropped when using only the node attributes. To answer this problem, we designed a learning to rank framework specific to the network data in Chapter 6.

In Chapter 4, we explained how to transfer knowledge from a hypertext encyclopedia to text analysis tasks. We have constructed a graph including Wikipedia articles and two different link structures between them. Our hypothesis was that using both word co-occurrence information and user-defined hyperlinks between articles could improve the resulting textual distance for application to a variety of tasks: word similarity; document similarity, clustering, and classification; information retrieval, and learning to rank.

To transfer learning from the Wikipedia network to text analysis tasks, we proposed and tested two representation methods. In the first one, a given text is mapped to the corresponding concepts in the network. Then, to compute similarity between two texts, *VP* similarity is applied to compute the distance between the two sets of nodes. In other words, the shared representation space is the set of concepts in the network and every text is represented in this space. The second method uses the latent space model explained in Chapter 3 as the shared representation, by training a transformation from words to the latent space over *VP* scores. Therefore, to transfer knowledge from the network to any machine learning algorithms, the given texts are transformed using this learned transformation. This second approach is easy to integrate with other machine learning algorithms and also speeds up similarity computation (testing time) in comparison to the first method.

We tested our proposals on different benchmark tasks: word similarity, document similarity, clustering, classification, information retrieval and learning to rank, and found that results were most often competitive compared to state-of-the-art task-specific methods. Our experimental results supported the hypothesis that both types of links over Wikipedia are useful, as the improvement of performance was higher when both were used together rather than separately. This sheds light on the fact that in many collaborative networks different link types can be used in a complementary way, and we investigated this problem formally in Chapter 5.

In Chapter 5, we proposed two joint similarity learning models over nodes' attributes for link prediction in networks with multiple link types. The first model learns a similarity metric that consists of two parts: the general part, which is shared between all link types, and the specific part, which is trained specifically for each type of link. The second model consists of two layers: the first layer, which is shared between all link types, embeds the objects of the network into a new space, and then a similarity is learned specifically for each link type in this new space.

Both models are applicable to large networks with high-dimensional feature spaces. The experiments showed that the proposed joint modeling and training frameworks improve link prediction performance significantly for each link type in comparison to multiple baselines. This improvement is higher when there are fewer links available from one link type in the network. The two-layer similarity model outperforms the first one, as expected, due to its capability of modeling negative correlations among different link types. Moreover, we illustrated that even if the models are trained on only one link type and tested on the other, our models significantly improve the performance in comparison to ad-hoc similarity metrics.

Finally, in Chapter 6, we proposed a learning to rank algorithm on network data, which uses both the attributes of the nodes and the structure of the links, for learning and inference. The global link structure of the network is used in inference by using an original propagation algorithm named IRA. This algorithm propagates the predicted scores in the graph on condition that they are above a given threshold. Thresholding improves performance, and makes a time-efficient implementation possible, for application to large scale graphs. These improvements were explained considering a structural property of many networks, to be specific that they are small-word ones.

The experimental results showed that using the IRA algorithm improved the performance of link prediction on three different network data sets, for binary graded scores (connected vs. not connected). The results showed more specifically that our neighbors-aware ranker, which uses content features and scores of the neighbors, has a higher performance than the content-only ranker, the neighbors-only ranker, Relational Topic Model and two linked-based similarity measures. Moreover, using the IRA in addition to the neighbors-aware ranker improved the performance even more.

Overall, we proposed and evaluated distance learning methods over **large** and **collaborative** networks with **high-dimensional** features, to be used for prediction tasks. These methods make use of the global structure of a network and the attributes of its nodes. We also showed how to make use of the correlation between various link types for transfer learning. Moreover, we showed how to leverage reliable content in these networks to perform text analysis tasks, improving the state-of-the-art performance.

While we showed the importance of similarity learning in analysis and modeling collaborative networks, we propose in what follows a perspective on other concepts that can also play role, which should be the topics of future investigations.

Perspectives on Similarity, Consistency and Diversity in Networks

Recommended sets of items generated by a recommender system are limited, in practice, to a few items. In this thesis, we almost always followed the assumption that the score assigned to such a set of items is equal to the sum of the scores of each item, given by its similarity with the query object. This assumption conforms to the homophily principle, but is in fact stronger than it. For instance, one exception was presented in Chapter 6, in which we considered the dependency between linked objects and showed improvement in the prediction performance, assuming also that there should be *consistency* between the scores of the linked objects.

Consistency increases the relevance of the recommended set by reducing the value of uncertain objects and increasing the value of coherent objects. This uncertainty is usually due to the imperfect content analysis techniques or intrinsic noise in the content of the query object. For example, in a recommender system for multimedia data, this uncertainty can be due to the imperfection of audio and video content analysis methods. Moreover, the uncertainty can be due to the intrinsic noise introduced by the unfocused nature of human communication, for example a video about social network analysis might also mention briefly the application of social network analysis to neuroscience.

In these cases, if we could also model *consistency* of the recommended set, then we would be able to reduce some of the uncertainty in the recommended set. In Chapter 4, Section 4.12, we showed that performing random walk on the network of objects implicitly solves this problem for a speech-based recommender system. In the following, we will sketch a formal definition of consistency based on the models presented in this thesis, to be explored in future work.

In almost all collaborative networks there is a cost for creating a link and therefore, the number of links per object is limited. For example, the number of products customers can buy from an online store is limited by their budget. The number of citations in a paper is limited because authors can not afford reading all papers. Therefore, forming a link between similar objects is more likely, but it does not imply that each object should necessarily connect to all similar objects.

Hence, among all similar objects, a *diverse* set of objects should be chosen for linking, so that it covers most aspects of the query object. For example, if an article is about the application of Hidden Markov Models to cryptanalysis, only important papers about Hidden Markov Models and cryptanalysis should be cited. *Diversity* thus plays a role in link formation in many examples, such as Wikipedia, news readers, article citations, consumer products, and music recommendation. Moreover, in difficult queries like ambiguous ones, diversification of the recommended set reduces the number of times that the system fails to recommending at least one relevant item.

Introducing Consistency and Diversity to the Latent Space Model

In this section, we will show how the latent space model which we used frequently in this thesis can be generalized to consider diversity and consistency, and indicate the investigation directions opened by this generalization.

If $S(q,T)$ is utility function represents the score assigned to the set T given the query q and x_i is the normalized feature vector of object i. Using the assumption of independence of objects and the latent space model we have:

$$S(q,T) = \sum_{i\in T}(x_q A)(x_i A)' = \sum_{i\in T}\sum_j (x_q A)_j (x_i A)_j = \sum_j \underbrace{\sum_{i\in T}(x_q A)_j (x_i A)_j}_{\text{reward of latent feature } j}$$

This is the linear model which sorts the objects according to their similarity to the query object in the latent space, and returns the first k objects regardless of the dependencies between them. $(x_i A)_j$ represents the "social characteristic" or latent feature j of the object i in the latent space and $(x_q A)_j (x_i A)_j$ represents how much the query object and object i are similar with respect to the feature j . In the future, we propose to generalize this model using the following formula which now includes parameter p as exponent:

$$S(q,T) = \sum_j \underbrace{(\sum_{i\in T}(x_q A)_j (x_i A)_j)^p}_{\text{reward of latent feature } j}$$

According to the value of p, we will be able to emphasize either of the following correlated aspects of the ranked set of objects.

Convexity ↔ supermodularity ↔ consistency: If $p = 1$, this model is equal to the previous linear model. If $p > 1$, i.e. the reward of a latent feature j is convex, then the reward of adding an object with latent feature j is higher when there are other objects with the same latent feature j. This model thus boosts *consistency* in the recommended set. In this case, the utility function $S(q,T)$ is supermodular for a given query and therefore, finding the set of objects with the highest score amounts to maximizing a supermodular function.

Concavity ↔ submodularity ↔ diversity: If $0 < p < 1$, i.e. the reward of the latent feature j is concave, then the reward of adding an object with the latent feature j is smaller when there are other objects with the same latent feature j in the set. In other words, the reward of adding an object is smaller if there are other similar objects in the set. In this case, the model gives a higher score to a *diverse* set of items. The utility function $S(q,T)$ is now submodular for a given query and therefore, finding the top score set of items amounts to maximizing a submodular function.

These proposals enrich the latent space model to encompass the concepts of diversity and consistency. The model still has the main desirable properties for the objective function: first, it gives a higher score to a set with items that are *related* to the query object (homophily) and, second, it emphasizes the *diversity* or *consistency* of items depending on the value of p.

There are several questions that must be investigated before putting this model into application. First, we need efficient algorithms to train both set of parameters, namely A and p. Moreover, is p dependent on the query object? It may be the case, indeed, that for some query objects diversity is more important than consistency, or vice-versa. In this case, can we learn the value of p depending on the query object? Besides, is p equal for all latent features or does it change for different features? Again, consistency could be more important for some features and diversity for other ones. Moreover, is it possible to consider both diversity and consistency simultaneously for a query object? It may be the case that given a query, a diverse set of consistent objects is the best recommended set.

Finally, after training these parameters, finding a set with a maximal score is not tractable for large data sets. In the case of a submodular utility function (with diversity being the target), efficient greedy algorithms have been proposed for inference on large data sets [Nemhauser et al., 1978, Minoux, 1978, Krause and Golovin, 2012]. Conversely, if the utility function is supermodular (consistency being the target), we need to design efficient algorithms to find the maximal set, or, presumably, approximate it. We foresee further investigations on these topics to make training and inference possible on large data sets.

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REPORTS

Majid Yazdani, Andrei Popescu-Belis. *Joint Similarity Learning for Predicting Links in Networks with Multiple-type Links.* Idiap-RR Idiap-Internal-RR-64-2012, Submitted , 2013.[.pdf]

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Work and Teaching Experiences

Work Experience:

- Postdoctoral Researcher: Building a toolbox for "Learning on large network data", Idiap research Institute , Martigny, Switzerland, March 2013- now
- Research Assistant : Idiap research Institute , Martigny, Switzerland, 2009- February 2013
- Research Assistant : Web Intelligence laboratory , Sharif University of Technology , 2007-2008

Teaching

- Co-Supervised a Master Semester Project on "topic modeling vs. semi-structured knowledge sources": EPFL, Switzerland, 2012
- Teaching Assistant : Computer Simulation Fall 2007, Mathematics Department, Sharif University of technology

COMPUTER AND LANGUAGE SKILLS

- Programming Languages: Proficient in java , MATLAB, R, C , Python, C++ , Lua, Prolog , Verilog , Motorolla 68000 assembly language, JSP , PHP, Django
- Simulation & Synthesis Environments: PGen (Parser Generator), Boson Network Designer, FPGA application software: QUARTUS II, NS2
- Data Base Management system: PostgreSQL, MySQL, MongoDB
- Languages : Persian (Mother tongue) , English (fluent) , Arabic (familiar/reading) , French (beginner)

Coursework

- **EPFL:** Machine Learning , Learning and Geometry , Statistical Sequence Processing (AUD), Probabilistic Graphical models (AUD) Computational linguistics , Advanced algorithms , Link Mining and Dynamic Network Analysis

- **Sharif University of Technology :** Computational Logic, AI Planning , Data Mining, Compiler Design , Artificial Intelligence, Computer Simulation, Advanced Information retrieval, Probability and statistics, Algorithm design

Micellaneous

- Human-Intelligence routing of questions in web: Human acquaintance network forms a small-world network, which can be used to find the community of experts for any question. BulbMeBack.com is an on-demand social network forming for each specific question to find the experts by using human intelligence routing of the questions (still progressing).

- Online Planning under Uncertainty Trading Algorithm: The online algorithm maximizes the benefit given the current portfolio and uncertain assumptions about the market. The algorithm has been running since 2011 on live data and showed promising results.
- One year studying chemistry at last year of high school, resulting in entrance to national Young Scholars Club and silver medal in national chemistry olympiad.