Speech power spectra: a window into neural oscillations in Parkinson's disease

Sevada Hovsepyan[1]*, Mathew Magimai.-Doss*[1]

[1]Idiap Research Institute, Martigny, Switzerland,

{sevada.hovsepyan, mathew}@idiap.ch

Abstract

This study analyzes the power spectral density (PSD) of speech in healthy controls (HC) and Parkinson's disease (PD) patients, focusing on the 0-100 Hz range. These low frequency components are below the fundamental frequency and may reflect both motor and neural mechanisms in speech production. We hypothesize that neural oscillations (NOs) involved in speech perception and production - theta (4-8 Hz), beta (15-35 Hz), and gamma (36-80 Hz) - shape the low-frequency PSD. Since NOs are linked to motor control and cognition, and are altered in PD, we expect systematic differences between HC and PD speech. Using multitaper estimation, we found significant differences in beta power, in line with research on beta oscillations and motor dysfunction in PD. Beyond distinguishing HC from PD speech, our results suggest that sub-fundamental frequency information may reflect neural dynamics in speech production, offering new perspectives for speech pathology and neural oscillation research.

Index Terms: Parkinson's disease, neural oscillations, beta oscillations, speech analysis, speech disorders

1. Introduction

Parkinson's disease (PD) is a common neurodegenerative disorder affecting millions of people worldwide [1, 2]. While it is primarily characterized by motor impairments such as tremor, bradykinesia, and motor planning difficulties [3], it also affects non-motor functions, including cognitive, gastrointestinal, and neuropsychiatric systems [4, 5]. These, both motor and non-motor, deficits can affect PD patients' speech production, often making their speech sound monotonous, disjointed, and emotionally flat [6].

In recent years, numerous studies have focused on the automatic detection of PD from speech [7, 8, 9]. Most of these studies have examined articulatory differences resulting from impaired motor control. However, speech changes in PD may also result from changes in brain activity - specifically, disruptions in neural oscillations that play a crucial role in speech production and perception [10, 11].

In this study, we analyze the power spectral density (PSD) of speech from Parkinson's disease patients and healthy controls (HC), focusing on frequency bands that correspond to neural oscillations involved in speech production [11, 12]. We then compare the power levels in these bands between the two conditions to see if there are systematic differences between PD and HC patients in terms of spectral power in frequency bands associated with NOs.

Our novel approach reveals traces of neural oscillations in produced speech, particularly in the power spectral density below 100 Hz, with notable peaks in the theta, beta, and gamma bands. Moreover, we also show statistically significant differences in these bands, especially in beta power, between healthy controls and PD patients. The remainder of this manuscript is organized as follows: in the **Background** section, we briefly summarize how PD, NOs, and speech perception/production are related, and then formulate the objectives of this study. The **Methods** section describes the computation of PSD from speech samples and the statistical tests performed. The **Results** section outlines our findings, and the **Discussion and Outlook** section summarizes our results, discusses their implications, and outlines limitations.

2. Background

2.1. Parkinson's disease and speech

The detection of PD in speech has been an active area of research, with studies examining various acoustic and articulatory features to identify characteristic speech impairments [9, 6]. While early research focused primarily on separating the source and system components of speech [13, 14], more recent work has expanded beyond production mechanisms to include perceptual aspects of speech [8, 15]. This includes the analysis of spectral features, that capture how the auditory system processes speech. For example in a recent study [15], the authors successfully used long- and short-term features based on Mel Frequency Cepstral Coefficients (MFCCs) for automatic PD detection from speech, achieving about 88 % accuracy. In another recent study, the authors compared different spectrogram methods (Short Term Fourier Transfer (STFT), MFCC, Cochleagrams) to construct syllable-level features to detect PD from speech [16]. The study showed that syllable nucleus based segmentation and STFT spectrogram lead to about 85% AUC PD detection from diadochokinetic speech.

2.2. Parkinson's disease and neural oscillations

Neural oscillations, or brain waves, refer to the rhythmic activity of the brain. In 1924, Hans Berger performed the first electroencephalogram (EEG) recording and discovered that brain activity is rhythmic [17]. Since then, neural oscillations (NOs) have been a major focus of neuroscience research and have been linked to various cognitive and mechanistic functions [18]. A key aspect of the neurophysiological changes associated with Parkinson's disease (PD) is the alteration of brain oscillatory activity. Specifically, PD has been associated with disruptions in beta oscillations ($13 - 35Hz$) [19] and altered cross-frequency coupling, such as beta-gamma interactions, associated with motor control [20, 21].

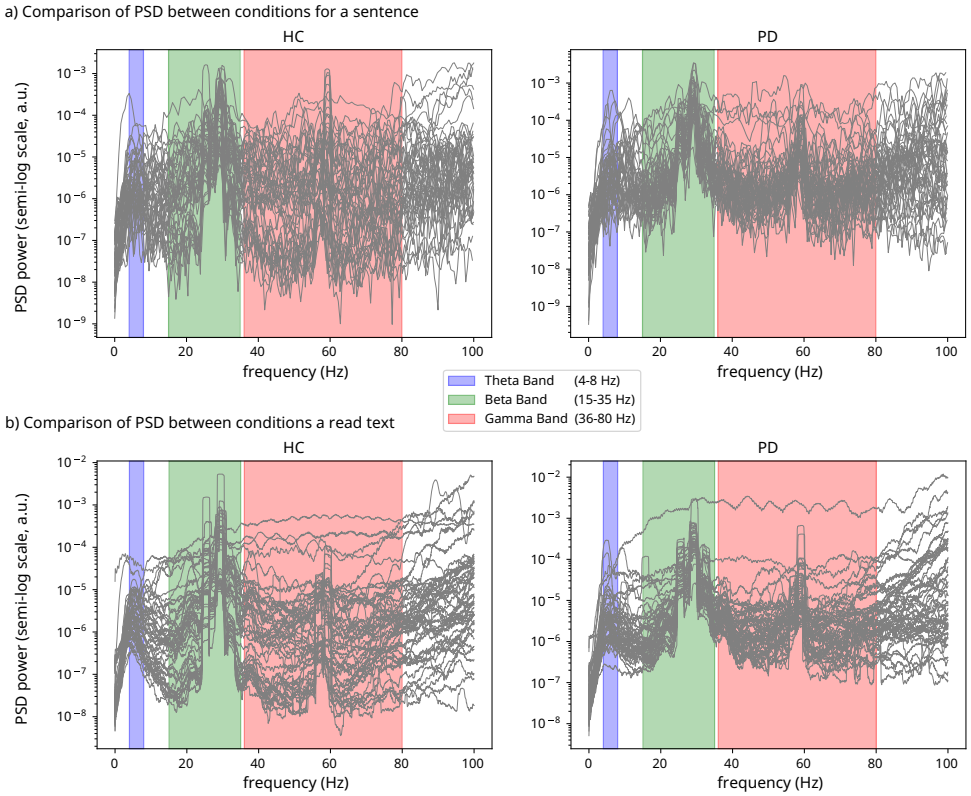

Figure 1: *PSD curves representing for each speaker (gray lines) in HC and PD condition for a) representative sentence task and b) Read text task*

2.3. Neural oscillations and speech

Traditionally, NOs have been grouped and named into specific bands and associated with different cognitive functions. Some of these bands, such as oscillations like theta ($4 - 8\ Hz$) and gamma ($30-80\ Hz$), operate on similar timescales as linguistic units in speech (e.g. syllables and phonemes/formant transitions respectively) [22, 12]. These similarities led to the hypothesis that brain oscillations are involved in speech processing and that different bands have different functions [12]. For example, theta is believed to be involved in syllable parsing, beta ($15-35\ Hz$) in motor coordination and speech timing, and gamma ($30 - 80\ Hz$) in phonemic encoding and sensory integration [23].

2.4. Research objectives and hypothesis

The previous sections have highlighted commonalities between Parkinson's disease and speech changes that may be due to motor problems or altered brain function and oscillations that affect speech. It remains unclear whether speech deficits in PD are due solely to impaired motor control, a direct result of disrupted neural oscillations related to speech processing, or a combination of both. The study hypothesizes that differences in low-frequency PSD between PD and healthy speech reflect underlying physiological mechanisms. By examining the spectral equivalents of theta, beta, and gamma neural oscillations, we aim to better understand the relationship between speech production and neural dynamics in PD.

3. Methods

3.1. Dataset

The objective of this study is to ascertain whether speech differences between healthy and pathological conditions reflect differences in brain activity. To this end, we have selected to utilize speech samples from the PC-GITA database [24], which includes Spanish-language recordings from 50 individuals with PD (25 male, 62.2 ± 11.2 years; 25 female, 60 .1 ± 7.8 years) and 50 healthy control (HC) individuals (25 male, 61.2 ± 11.3 years; 25 female, 60.7 ± 7.7 years). The recordings are characterized by their clarity and were captured using a professional audio card with up to 24 bits and 96 kHz sampling rate (M-Audio, Fast Track C400) in noise-controlled conditions within a soundproof booth. However, the recordings in the dataset are provided at 16-bit resolution and 44.1 kHz sampling rate. All patients were diagnosed by neurologist experts and labeled according to the UPDRS [25] and H&Y [26] scales. These recordings encompass several distinct speech types, such as: diadochokinetic (DDK) utterances, monologues, read text, and sentences, etc. As the objective of this study is to identify neural oscillatory traces within the speech spectrum, we have prioritized speech samples that meet specific criteria. Namely, we have focused on samples that (1) involve cognitive load and (2) are sufficiently long to sustain oscillatory activity. Therefore, recording of sentences in the dataset satisfy these criteria and are thus considered suitable candidates for our analysis. Consequently, we selected six sentences from the PC-GITA dataset (coded with single words, as shown in Table 1 and Table 2) and recordings of "read text" speech type, which we used as a longer

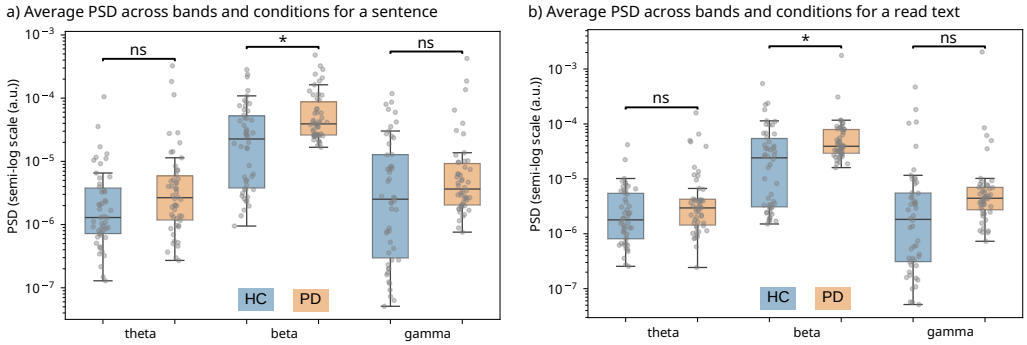

Figure 2: *Comparison of average power in the theta, beta, and gamma bands across conditions (HC-blue vs. PD-red) for sentence (a) and read speech (b) recordings. Horizontal lines above the box plots indicate the significance of the Mann-Whitney U-test (asterisks indicate that the median power in the respective bands is statistically different).*

speech sample.

3.2. Band power calculations

To compute the power spectral density (PSD) of speech samples with higher resolution at low frequencies, we used the multitaper method [27] as implemented in the Minimum Norm Estimates (MNE) library [28] (specifically, `psd_array_multitaper` with a frequency range of 0-100 Hz and a bandwidth of 2 Hz). The PSD for each speaker, across conditions and speech types, is shown as gray lines in Figure 1.

To quantify power within specific frequency bands, we calculated the area under the PSD curve for each band of interest and normalized it to the bandwidth. The resulting values, shown in the box plots in Figure 2, were used for the statistical analyses presented in Tables 1 and 2.

3.3. Statistical analysis

The mean power distributions for the bands of interest (theta, beta, gamma) in each condition were compared statistically using two tests: the Mann-Whitney U test for median differences and the Kolmogorov-Smirnov test for overall distributions. These tests were performed separately for each utterance, and the results are presented in Table 2. As we assessed differences in each band across conditions, significance was determined using a Bonferroni-corrected threshold of $\alpha = 0.05/3$.

3.4. Band correlation between conditions

We also examined the relationship between frequency bands within each condition using Spearman's correlation test to assess whether power in different bands are independent from each other. Therefore, for each band pair (theta-beta, theta-gamma, beta-gamma), the correlation coefficient (rho) was compared across conditions. To quantify these differences, the correlation coefficients were transformed using Fisher's Z-transformation and the resulting values were statistically evaluated using a Bonferroni-corrected $\alpha = 0.05/3$. The results provide insight into whether power in different bands (and thus corresponding neural oscillations) show an altered relationship in pathological speech.

4. Results

In this section, we present the power spectral density (PSD) analysis of speech samples from healthy controls (HC) and individuals with Parkinson's disease (PD) in the PC-GITA dataset [24]. We analyzed recordings of six individual sentences and one read text, but for clarity the figures and tables show results from one representative sentence ('*Mi casa tiene tres cuartos.*') and the Read Text recording. However, figures for other sentences, including both PSD analysis and box plots of average power in the bands of interest, are provided as supplemental information [1].

4.1. Inter-speaker variability in PSD patterns in healthy controls

The power spectral density was computed using the multitaper method to achieve high-frequency resolution, particularly in the lower frequencies. Figure 1 illustrates the PSD for each speaker (gray lines) in both conditions, with frequency bands of interest highlighted in different colors.

In the figure subpanels for sentence (a) and read text (b) recordings, characteristic peaks are evident in specific frequency regions for both single-sentence and multi-sentence scenarios. These peaks align with frequency bands typically associated with neural oscillations (as color-coded in the figure). Notably, greater variability is observed in the healthy condition for both speech types, whereas the Parkinson's disease condition exhibits less variability.

4.2. High beta power in Parkinson's disease condition

To quantify this information, we calculated the average power within each band (area under the curve divided by bandwidth) for each speaker. We then compared whether the resulting distributions for each band (separately for each speech utterance) were statistically different. We used the Mann-Whitney U-test for comparisons of medians, while we used the Kolmogorov-Smirnov (KS) test to compare overall distributions. Figure 2 shows the resulting scatter plots and box plots for the same utterances as in Figure 1, while Table 1 presents the detailed results of the statistical tests for all sentences and the "Read Text" recordings.

The figure show scatter plots and box plots illustrating the average PSD within each frequency band. The color of the box plots distinguishes between PD and HC conditions. The plots also show the results of statistical tests comparing the medians of the average PSD of each band across conditions. The significance level is set at $alpha = 0.05/0.03$ adjusted for multiple

[1] `https://doi.org/10.5281/zenodo.15516873`

Table 1: *Results of statistical analysis, p-values are corrected for multiple comparisons with the Bonferroni procedure*

speech type	band	u-stat	p-value (corr)	ks-stat	p-avlue (corr)
ReadText	theta	453	0.07556	0.22	0.17859
	beta	328	**0.00236**	0.46	**0.00004**
	gamma	472	0.11172	0.42	**0.00025**
laura	theta	529	0.29995	0.3	**0.02171**
	beta	356	**0.00594**	0.44	**0.00010**
	gamma	464	0.09513	0.4	**0.00058**
loslibros	theta	483	0.13807	0.26	0.06779
	beta	277	**0.00034**	0.42	**0.00025**
	gamma	384	**0.01369**	0.42	**0.00025**
luisa	theta	479	0.12799	0.28	**0.03919**
	beta	382	**0.01293**	0.44	**0.00010**
	gamma	476	0.12081	0.38	**0.00131**
micasa	theta	462	0.09131	0.24	0.11239
	beta	303	**0.00095**	0.44	**0.00010**
	gamma	609	0.78875	0.38	**0.00131**
omar	theta	385	**0.01408**	0.28	**0.03919**
	beta	240	**0.00007**	0.44	**0.00010**
	gamma	576	0.55910	0.42	**0.00025**
rosita	theta	463	0.09320	0.28	**0.03919**
	beta	308	**0.00115**	0.46	**0.00004**
	gamma	463	0.09320	0.44	**0.00010**

comparisons. The results for the other sentence types and the KS test are summarized in Table 1.

As shown in Table 1, all sentence types show a statistically significant difference in beta band power between HC and PD conditions, supported by both the KS-test and Mann-Whitney U-test. While some bands show additional significant differences in some sentences, only the beta band is consistently different between conditions.

These findings confirm our earlier observation of greater variability in the HC condition compared to the narrower spread in the PD condition. These results suggest that beta band activity is more concentrated and/or more powerful in PD patients.

4.3. Reduced correlation of band powers in Parkinson's disease

Finally, we examined whether different frequency bands were dependent on each other by calculating Spearman correlation coefficients between their power values within each condition. For each pair of bands, we then compared the correlation coefficients between conditions (after applying Fisher's z-transformation) and tested for statistical significance. The results of these analyses are summarized in Table 2

As shown in Table 2, the correlation between the average power of the frequency bands is relatively high in the HC condition, whereas it is generally low in the PD condition, except for the theta-gamma power correlation, which remains somewhat high. Furthermore, the differences between the correlation coefficients are statistically significant for all beta-gamma correlations and most of the theta-beta correlations. These results reinforce the earlier observation that beta-band power behaves differently in the PD condition (as indicated by the statistically significant differences between conditions in Table 1). In addition, these results suggest that power in these bands is largely independent (or weakly dependent) in the PD condition, whereas in HC condition bands are more interrelated.

Table 2: *Statistical comparison of correlation coefficients between conditions*

speech type	band (HC)	band (PD)	rho (HC)	rho (PD)	z-diff	p-value (corr)
ReadText	**theta**	**beta**	**0.694**	**0.066**	**3.829**	**3.86E-04**
	theta	**gamma**	**0.724**	**0.222**	**3.344**	**2.48E-03**
	beta	**gamma**	**0.853**	**0.283**	**4.738**	**6.49E-06**
laura	**theta**	**beta**	**0.675**	**0.160**	**3.194**	**4.20E-03**
	theta	gamma	0.610	0.329	1.777	2.27E-01
	beta	**gamma**	**0.819**	**0.225**	**4.485**	**2.18E-05**
loslibros	**theta**	**beta**	**0.560**	**-0.105**	**3.580**	**1.03E-03**
	theta	gamma	0.562	0.258	1.804	2.13E-01
	beta	**gamma**	**0.838**	**0.083**	**5.479**	**1.28E-07**
luisa	**theta**	**beta**	**0.643**	**0.059**	**3.418**	**1.89E-03**
	theta	gamma	0.672	0.388	1.960	1.50E-01
	beta	**gamma**	**0.850**	**0.343**	**4.351**	**4.07E-05**
micasa	theta	beta	0.462	0.150	1.693	2.71E-01
	theta	gamma	0.500	0.356	0.856	1.00E+00
	beta	**gamma**	**0.887**	**0.276**	**5.445**	**1.55E-07**
omar	**theta**	**beta**	**0.583**	**-0.155**	**3.987**	**2.01E-04**
	theta	gamma	0.569	0.232	1.987	1.41E-01
	beta	**gamma**	**0.847**	**0.309**	**4.497**	**2.06E-05**
rosita	theta	beta	0.610	0.301	1.927	1.62E-01
	theta	**gamma**	**0.626**	**0.143**	**2.862**	**1.26E-02**
	beta	**gamma**	**0.844**	**0.408**	**3.890**	**3.01E-04**

5. Discussion and Outlook

In this study, we aimed to detect traces of neural oscillations in speech by analyzing the speech power spectral density in the lower frequency range (0-100 Hz). Our hypothesis was that if neural oscillations influence speech production, their effects would be evident in the sub-fundamental frequency region of the PSD. We further hypothesized that comparing speech from healthy individuals to speech from individuals with Parkinson's disease (PD), a condition known to alter neural oscillations, would enhance our ability to reveal these traits.

Our analysis revealed peaks in the PSD corresponding to the frequency bands of cortical theta, beta, and gamma oscillations. In furthermore, we found systematic differences in beta power between the two conditions, with significantly higher beta power in PD speech in all analyzed speech samples. This finding is consistent with neurophysiological evidence of altered beta oscillatory activity in PD patients [29, 30].

While our results suggest that sub-fundamental frequency information in speech may be a reflection of underlying neural activity, further research is necessary for confirmation. Direct comparisons with brain recordings and validation with additional data sets are necessary to establish a direct link between speech PSD and brain activity. Nevertheless, our findings highlight the potential of speech analysis to provide insight into pathological speech and neural oscillations, opening new avenues for research and clinical applications. It is worth noting that the current study serves as a proof of principle. In future work, we plan to further explore these findings by (e.g.) examining PSD patterns across conditions, rather than focusing solely on average power in specific bands.

6. Acknowledgements

This work was funded by the Swiss National Science Foundation through the Bridge Discovery project EMIL: Emotion in the loop - a step towards a comprehensive closed-loop deep brain stimulation in Parkinson's disease (grant no. 40B2–0_194794).

7. References

[1] M. C. de Rijk, L. J. Launer, K. Berger, M. M. Breteler, J. F. Dartigues, M. Baldereschi, L. Fratiglioni, A. Lobo, J. Martinez-Lage, C. Trenkwalder, and A. Hofman, "Prevalence of Parkinson's disease in Europe: A collaborative study of population-based cohorts. Neurologic Diseases in the Elderly Research Group," *Neurology*, vol. 54, no. 11 Suppl 5, pp. S21–23, 2000.

[2] A. Samii, J. G. Nutt, and B. R. Ransom, "Parkinson's disease," *Lancet (London, England)*, vol. 363, no. 9423, pp. 1783–1793, May 2004.

[3] M. S. Barnish and S. M. Barran, "A systematic review of active group-based dance, singing, music therapy and theatrical interventions for quality of life, functional communication, speech, motor function and cognitive status in people with Parkinson's disease," *BMC Neurology*, vol. 20, p. 371, Oct. 2020.

[4] R. F. Pfeiffer, "Non-motor symptoms in Parkinson's disease," *Parkinsonism & Related Disorders*, vol. 22, pp. S119–S122, Jan. 2016.

[5] P. Mazzoni, B. Shabbott, and J. C. Cortés, "Motor Control Abnormalities in Parkinson's Disease," *Cold Spring Harbor Perspectives in Medicine*, vol. 2, no. 6, p. a009282, Jun. 2012.

[6] A. Rohl, S. Gutierrez, K. Johari, J. Greenlee, K. Tjaden, and A. Roberts, "Speech dysfunction, cognition, and Parkinson's disease," *Progress in Brain Research*, vol. 269, no. 1, pp. 153–173, 2022.

[7] R. Gupta, S. Kumari, A. Senapati, R. K. Ambasta, and P. Kumar, "New era of artificial intelligence and machine learning-based detection, diagnosis, and therapeutics in Parkinson's disease," *Ageing Research Reviews*, vol. 90, p. 102013, Sep. 2023.

[8] M. G. Di Cesare, D. Perpetuini, D. Cardone, and A. Merla, "Machine Learning-Assisted Speech Analysis for Early Detection of Parkinson's Disease: A Study on Speaker Diarization and Classification Techniques," *Sensors*, vol. 24, no. 5, p. 1499, Jan. 2024, number: 5 Publisher: Multidisciplinary Digital Publishing Institute. [Online]. Available: https://www.mdpi.com/1424-8220/24/5/1499

[9] A. Bayestehtashk, M. Asgari, I. Shafran, and J. McNames, "Fully Automated Assessment of the Severity of Parkinson's Disease from Speech," *Computer Speech & Language*, vol. 29, no. 1, pp. 172–185, Jan. 2015.

[10] W. Cai, C. B. Young, R. Yuan, B. Lee, S. Ryman, J. Kim, L. Yang, T. F. Levine, V. W. Henderson, K. L. Poston, and V. Menon, "Subthalamic nucleus–language network connectivity predicts dopaminergic modulation of speech function in Parkinson's disease," *Proceedings of the National Academy of Sciences*, vol. 121, no. 22, p. e2316149121, May 2024.

[11] L. Meyer, "The neural oscillations of speech processing and language comprehension: State of the art and emerging mechanisms," *European Journal of Neuroscience*, vol. 48, no. 7, pp. 2609–2621, Nov. 2018.

[12] A. L. Giraud and D. Poeppel, "Cortical oscillations and speech processing: Emerging computational principles and operations," *Nature Neuroscience*, vol. 15, no. 4, pp. 511–517, Mar. 2012. [Online]. Available: http://www.nature.com/doifinder/10.1038/nn.3063

[13] S. P. Dubagunta, B. Vlasenko, and M. Magimai.-Doss, "Learning Voice Source Related Information for Depression Detection," in *ICASSP 2019 - 2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. Brighton, United Kingdom: IEEE, May 2019, pp. 6525–6529.

[14] F. Amato, L. Borzì, G. Olmo, and J. R. Orozco-Arroyave, "An algorithm for Parkinson's disease speech classification based on isolated words analysis," *Health Information Science and Systems*, vol. 9, no. 1, p. 32, Jul. 2021. [Online]. Available: https://doi.org/10.1007/s13755-021-00162-8

[15] S. Hawi, J. Alhozami, R. AlQahtani, D. AlSafran, M. Alqarni, and L. E. Sahmarany, "Automatic Parkinson's disease detection based on the combination of long-term acoustic features and Mel frequency cepstral coefficients (MFCC)," *Biomedical Signal Processing and Control*, vol. 78, p. 104013, Sep. 2022.

[16] S. Hovsepyan and M. Magimai.-Doss, "Syllable Level Features for Parkinson's Disease Detection from Speech," in *ICASSP 2024 - 2024 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. Seoul, Korea, Republic of: IEEE, Apr. 2024, pp. 11 416–11 420.

[17] H. Berger, "Über das Elektrenkephalogramm des Menschen," *Deutsche Medizinische Wochenschrift*, vol. 60, no. 51, pp. 1947–1949, 1929.

[18] K. B. Doelling and M. F. Assaneo, "Neural oscillations are a start toward understanding brain activity rather than the end," *PLoS Biology*, vol. 19, no. 5, p. e3001234, May 2021.

[19] V. Mathiopoulou, R. Lofredi, L. K. Feldmann, J. Habets, N. Darcy, W.-J. Neumann, K. Faust, G.-H. Schneider, and A. A. Kühn, "Modulation of subthalamic beta oscillations by movement, dopamine, and deep brain stimulation in Parkinson's disease," *npj Parkinson's Disease*, vol. 10, no. 1, pp. 1–7, Apr. 2024.

[20] C. de Hemptinne, E. S. Ryapolova-Webb, E. L. Air, P. A. Garcia, K. J. Miller, J. G. Ojemann, J. L. Ostrem, N. B. Galifianakis, and P. A. Starr, "Exaggerated phase–amplitude coupling in the primary motor cortex in Parkinson disease," *Proceedings of the National Academy of Sciences*, vol. 110, no. 12, pp. 4780–4785, Mar. 2013.

[21] T. Hodnik, S. Roytman, N. I. Bohnen, and U. Marusic, "Beta-Gamma Phase-Amplitude Coupling as a Non-Invasive Biomarker for Parkinson's Disease: Insights from Electroencephalography Studies," *Life (Basel, Switzerland)*, vol. 14, no. 3, p. 391, Mar. 2024.

[22] G. Mai, J. W. Minett, and W. S. Wang, "Delta, theta, beta, and gamma brain oscillations index levels of auditory sentence processing," *NeuroImage*, vol. 133, pp. 516–528, Jun. 2016. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1053811916001737?via%3Dihub

[23] L. H. Arnal and A. L. Giraud, "Cortical oscillations and sensory predictions," p. 398, Jul. 2012. [Online]. Available: http://linkinghub.elsevier.com/retrieve/pii/S1364661312001210

[24] J. R. Orozco, J. D. Arias-Londoño, J. Vargas-Bonilla, M. González-Rátiva, and E. Noeth, "New Spanish speech corpus database for the analysis of people suffering from Parkinson's disease," in *International Conference on Language Resources and Evaluation (LREC)*, May 2014.

[25] G. T. Stebbins and C. G. Goetz, "Factor structure of the Unified Parkinson's Disease Rating Scale: Motor Examination section," *Movement Disorders: Official Journal of the Movement Disorder Society*, vol. 13, no. 4, pp. 633–636, Jul. 1998.

[26] M. M. Hoehn and M. D. Yahr, "Parkinsonism: Onset, progression and mortality," *Neurology*, vol. 17, no. 5, pp. 427–442, May 1967.

[27] D. Slepian, "Prolate spheroidal wave functions, fourier analysis, and uncertainty — v: the discrete case," *The Bell System Technical Journal*, vol. 57, no. 5, pp. 1371–1430, 1978.

[28] A. Gramfort, M. Luessi, E. Larson, D. A. Engemann, D. Strohmeier, C. Brodbeck, R. Goj, M. Jas, T. Brooks, L. Parkkonen, and M. S. Hämäläinen, "MEG and EEG data analysis with MNE-Python," *Frontiers in Neuroscience*, vol. 7, no. 267, pp. 1–13, 2013.

[29] N. Morelli and R. L. S. Summers, "Association of subthalamic beta frequency sub-bands to symptom severity in patients with Parkinson's disease: A systematic review," *Parkinsonism & Related Disorders*, vol. 110, p. 105364, May 2023. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1353802023000871

[30] Y. Yu, D. Escobar Sanabria, J. Wang, C. M. Hendrix, J. Zhang, S. D. Nebeck, A. M. Amundson, Z. B. Busby, D. L. Bauer, M. D. Johnson, L. A. Johnson, and J. L. Vitek, "Parkinsonism Alters Beta Burst Dynamics across the Basal Ganglia–Motor Cortical Network," *The Journal of Neuroscience*, vol. 41, no. 10, pp. 2274–2286, Mar. 2021. [Online]. Available: https://www.ncbi.nlm.nih.gov/pmc/articles/PMC8018776/